Chapter 2

Polynomials

In the Chapter

- **(a) Algebra :** The branch of mathematics which deals with variables and four fundamental operations on them, is called Algebra.
- **Variable :** A variable is a number which can have different values whereas a constant has a fixed value.
- **Algebraic Expression :** An algebraic expression is a number or a combination of numbers including variable joined by the four fundamental operations.
- **Term :** When one or more of the symbols + or – occur in an algebraic expression, they separate the algebraic expression into parts, each of the which, is called term.
  A term contains either a variable or a constant or both variable (s) and constant connected by the operating of multiplication .
- **(b) Polynomials :** An algebraic expression in which the variable(s) does (do) not occur in the denominator and the exponents of the variable (or variables) are whole numbers and the co-efficients of different terms are real numbers is called a **polynomial.**
- **General form of a polynomial in one variable :** An expression of the form

$$a_0 + a_1x + a_2x^2 + ... + a_nx^n$$

where a_0 , a_1, a_2 ... a_n are real constants and $a_n \neq 0$. x is a variable and n is a non-negative integer is called polynomial in the variable x.
- **Zero Polynomial :** If $a_0 = a_1 = a_2 = ... = a_n = 0$, (i.e., all constants are zeros then the polynomials reduces to zero which is called zero polynomial.
- **Degree of a polynomial :** The degree of a polynomial in one variable is the greatest exponent of the variable occurring in the various terms of the polynomial.
  The degree of a constant is taken as zero. A polynomial in one variable is written in decreasing power of the variable and this is called the **standard form** of the polynomial.
- **Various types of polynomials :**
  (a) **Linear Polynomial :** A polynomial of degree one is called a linear polynomial.
  e.g.,
  $x + \sqrt{7}$ is a linear polynomial in x.
  $\sqrt{4}\,\mu + 3$ is a linear polynomial in μ.
  (b) **Quadratic Polynomial :** A polynomial of degree two is called a quardratic polynomial.
  e.g.,
  $xy + yx + zx$ is a quadratic polynomial in x, y, and z.
  $x^2 + 9x - \frac{3}{2}$ is a quadratic polynomial in x.
  (c) **Cubic Polynomial** : A polynomial of degree three is called a cubic polynomial.
  e.g.,
  $ax^3 + bx^2 + cx + d$ is a cubic polynomial in x and a, b, c, d are constants.

$2y^3 + 3$ is a cubic polynomial in y.

$9x^2y + xy - 4$ is a cubic polynomial in x and y.

(d) **Binomial :** Polynomials having only two terms are called binomials ('bi' means 'two')

e.g.

$(x^2 + x)$, $(y^{30} + \sqrt{2})$ and $(5x^2y + 6xz)$ are all binomials.

(e) **Trinomial :** Polynomials having only three terms are called trinomials ('tri' means 'three').

e.g.,

$(x^4 + x^3 + \sqrt{2})$, $(\mu^{43} + \mu^7 + \mu)$ and $(8y - 5xy + 9xy^2)$ are all trinomials.

- **Factors and Multiples :**

  Factors : Factors of a polynomials are the polynomials whose products is the given polynomial.

  Example : If $x^2 + 5x + 6 = (x + 2)(x + 3)$, then $(x + 2)$ and $(x + 3)$ are factors of $x^2 + 5x + 6$.

  Multiples : Multiples of a polynomial are also the polynomials whose multiplicand is the given polynomials.

  Example : If $x^2 + 7x + 12 = (x + 3)(x + 4)$, then $(x + 3)$ and $(x + 4)$ are factors of $x^2 + 7x + 12$.

- **Factor Theorem :** Let $q(x)$ be a polynomial of degree $n > 1$ and a be any real number, then

  (i) $(x - a)$ is a factor of $q(x)$, if $q(a) = 0$ and

  (ii) $q(a) = 0$, if $x - a$ is a factor of $q(x)$

  Example : Examine whether $(x + 2)$ is a factor of $x^3 + 3x^2 + 5x + 6$.

  Sol. $p(x) = x^3 + 3x^2 + 5x + 6$

  $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6 = -8 + 12 - 10 + 6 = 0$

  Hence by factor theorem $(x + 6)$ is a factor of $x^3 + 3x^2 + 5x + 6$.

- **Zeroes of a Polynomial :** Zero of a polynomial $p(x)$ is a number a such that $p(a) = 0$.

  (a) Zero may be a zero of a polynomial.

  (b) Every linear polynomial has one and only one zero.

  (c) Zero of a polynomial is also called the root of polynomial.

  (d) A non-zero constant polynomial has no zero.

  (e) Every real number is a zero of the zero polynomial.

  (f) A polynomial can have more than one zero.

  Maximum number of zeroes of a polynomial is equal to its degree.

- **Remainder Theorm :** Let $p(x)$ be any polynomial of degree n greater than or equal to one $(n > 1)$ and let a be any real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$.

  Dividend = (Divisor × Quotient) + Remainder

- **Algebraic Identities :** An algebraic identity is an algebraic equation that is true for all values of the variable occurring in it.

  Some algebraic identities are given below

  (i) $(x + y)^2 = x^2 + 2xy + y^2$

  (ii) $(x - y)^2 = x^2 - 2xy + y^2$

  (iii) $x^2 - y^2 = (x + y)(x - y)$

  (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

  (v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

  (vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

  (vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

  (viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

  If $x + y + z = 0$, then $x^3 + y^3 + x^3 = 3xyz$

EXERCISE 2.1

Q.1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + \sqrt{2}t$ (iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Ans. (i) $4x^2 - 3x + 7$

It is a polynomial in one variable x only.

(ii) $y^2 + \sqrt{2}$

It is also a polynomial in one variable y.

(iii) $3\sqrt{t} + \sqrt{2}t$

It is not polynomial in one variable t, since the degree of t is 1/2.

(iv) $y + \frac{2}{y}$

It is also not a polynomal in one variable y since the degree of y is – 1.

(v) $x^{10} + y^3 + t^{50}$

It is not a polynomial in one variable as it involves x, y, and t.

Q.2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Ans. (i) In $2 + x^2 + x$, the coefficient of x^2 is 1.

(ii) In $2 - x^2 + x^3$, the coefficient of x^2 is –1.

(iii) In $\frac{\pi}{2}x^2 + x$, the coeffecient of x^2 is $\frac{\pi}{2}$.

(iv) in $\sqrt{2}x - 1$, the coefficient of x^2 is zero.

Q.3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Ans. (i) A binomial of degree 35 is $x^{35} + x$.

(ii) A monomial of degree 100 is $5y^{100}$.

Q.4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

(iii) $5t - 7$ (iv) 3

Ans. (i) Degree of $p(x) = 3$.

(ii) Degree of $p(y) = 2$.

(iii) Degree of $f(t) = 1$.

(iv) Degree of $f(x) = 0$.

Q. 5. Classify the following as linear, quadratic and cubic polynomials :

(i) $x^2 + x$ (ii) $x - x^3$

(iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) r^2

(vii) $7x^3$

Ans. (*a*) Linear polynomials are :

(iv) $1 + x$, (v) $3t$ [degree = 1]

(b) Quadratic polynomials are :

(i) $x^2 + x$, (iii) $y + y^2 + 4$, (vi) r^2 [degree = 2]

(c) Cubic polynomials are :

(ii) $x - x^3$, (vii) $7x^3$ [degree = 3]

EXERCISE 2.2

Q.1. Find the value of the polynomial $5x - 4x^2 + 3$ at :

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Ans. (i) $p(x) = 5x - 4x^2 + 3$ at $x = 0$.

$\therefore$ $p(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3 = 3$.

(ii) $p(x) = 5x - 4x^2 + 3$ at $x = -1$

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3 = -6$.

(iii) $p(x) = 5x - 4x^2 + 3$ at $x = 2$.

$p(2) = 5 \times 2 - 4(2)^2 + 3$

$= 10 - 16 + 3 = -3$.

Q.2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x - 1)(x + 1)$

Ans. (i) $p(y) = y^2 - y + 1$

$\therefore$ $p(0) = (0)^2 - 0 + 1 = 1$

$p(1) = (1)^2 - 1 + 1 = 1$

$p(2) = (2)^2 - 2 + 1$

$= 4 - 2 + 1 = 3$.

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$\therefore$ $p(0) = 2 + 0 + 2 \times (0)^2 - (0)^3 = 3$.

$p(1) = 2 + 1 + 2(1)^2 - (1)^3$

$= 3 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2(2)^2 = 4 + 8 = 12$.

(iii) $p(x) = x^3$

$\therefore$ $p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8.$

(iv) $p(x) = (x-1)(x+1)$
$p(0) = (0-1)(0+1) = 1$
$p(1) = (1-1)(1+1) = 0 \times 2 = 0$
$p(2) = (2-1)(2+1) = 1 \times 3 = 3.$

Q.3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = \frac{-1}{3}$

(ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$

(iv) $p(x) = (x+1)(x-2), x = -1, 2$

(v) $p(x) = x^2, x = 0$

(vi) $p(x) = lx + m, x = \frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, x = \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Ans. (i) $p(x) = 3x + 1, x = \frac{-1}{3}$

$\therefore \quad p\left(-\frac{1}{3}\right) = 3\times\left(-\frac{1}{3}\right)+1$
$= -1 + 1 = 0$

As $p(x) = 0$ for $x = -\frac{1}{3}$

$\therefore \quad x = -\frac{1}{3}$

is a zero of $p(x)$.

(ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

$\therefore \quad p\left(\frac{4}{5}\right) = 5 \times \frac{4}{5} - \pi = 4 - \pi$

As $p(x) = 0$ for $x = \frac{\pi}{5}$

Therefore $x = \frac{\pi}{5}$ is a zero of the given polynomial *p(x)*.

(iii) $p(x) = x^2 - 1, x = 1, -1$
$p(x) = x^2 - 1$
$x = 1, -1$
for $x = 1$
$p(1) = (1)^2 - 1 = 0$
and for $x = -1$
$p(-1) = (-1)^2 - 1 = 0$
As $p(x) = 0$ for $x = 1$
and $x = -1$
Therefore $x = 1, -1$
are zeroes of the given polynomial.
$p(x) = x^2 - 1$

(iv) $p(x) = (x+1)(x-2),$
$x = -1, 2$
for $x = -1$
$p(-1) = (-1+1)(-1-2) = 0$
for $x = 2$
$p(2) = (2+1)(2-2) = 0$

Hence x = –1, 2 are zeroes of the given polynomials.

$p(x) = (x+1)(x-2)$

(v) $p(x) = x^2, x = 0$
For $x = 0$
$p(0) = (0)^2 = 0$
Hence, $x = 0$ is a zero of the given polynomial
$p(x) = x^2$

(vi) $p(x) = lx + m,$
$x = \frac{m}{l}$

For $x = -\frac{m}{l}$

$p\left(-\frac{m}{l}\right) = l \times \left(-\frac{m}{l}\right) + m$
$= -m + m = 0$

Hence for $x = -\frac{m}{l}$, $p(x) = 0$

Therefore $x = -\frac{m}{l}$

is a zero of the given polynomial.

(vii) $p(x) = 3x^2 - 1, x = \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

$\therefore \quad p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$= 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$= 3 \times \frac{4}{3} - 1 = 3$

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

$p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} + 1$

$= 1 + 1 = 2.$

No, $x = \frac{1}{2}$ is not a zero of

$p(x) = 2x + 1$

Q.4. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$
(ii) $p(x) = x - 5$
(iii) $p(x) = 2x + 5$
(iv) $p(x) = 3x - 2$
(v) $p(x) = 3x$
(vi) $p(x) = ax, a \neq 0$
(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Ans. (i) $p(x) = x + 5$

$x + 5 = 0,$

$x = -5,$

Therefore, – 5 is the zero of $x + 5$.

(ii) $p(x) = x - 5$

$x - 5 = 0$

$x = 5.$

Therefore, 5 is the zero of $x - 5$.

(iii) $p(x) = 2x + 5$

$2x + 5 = 0$

$\Rightarrow \quad 2x = -5$

$\Rightarrow \quad x = \frac{-5}{2}$

Therefore, $\frac{-5}{2}$ is the zero for given polynomial $2x + 5$.

(iv) $p(x) = 3x - 2$

$3x - 2 = 0$

$\Rightarrow \quad 3x = 2$

$\Rightarrow \quad x = \frac{2}{3}$

Therefore, $\frac{2}{3}$ is the zero of $3x - 2$.

(v) $p(x) = 3x$

$3x = 0,$

$\Rightarrow \quad x = 0,$

Therefore, 0 is the zero of $3x$.

(vi) $p(x) = ax, a \neq 0$

$ax = 0 \ (a \neq 0)$

$\Rightarrow \quad x = \frac{0}{a} = 0.$

Therefore, 0 is the zero of ax.

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

$cx + 4 = (c \neq 0)$

$\Rightarrow \quad cs = -d$

$\Rightarrow \quad x = \frac{-d}{c}$

Therefore, $\frac{-d}{c}$ *is the zero of* cx + d.

EXERCISE 2.3

Q.1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x
(iv) $x + \pi$ (v) $5 + 2x$

Ans.(i) $x + 1$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x + 1 = 0$

$\Rightarrow \quad x = -1$

For $x = -1$

$f(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

Hence remainder $= 0$

(ii) $x-\frac{1}{2}$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x - \frac{1}{2} = 0$

$\Rightarrow \quad x = \frac{1}{2}$

$\therefore \quad f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$

$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1=\frac{27}{8}$

Hence remainder = $\frac{27}{8}$

(iii) x

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x = 0$

$\therefore \quad f(0) = (0)^3 + 3(0)^3 + 3 \times 0 + 1$

$= 1$

Hence remainder = 1

(iv) $x + \pi$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x + \pi = 0$

$\Rightarrow \quad x = -\pi$

$f(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3 \times (-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

Hence remainder = $-\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $5 + 2x = 0$

$\Rightarrow \quad x = -\frac{5}{2}$

$\therefore \quad f\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$

$= -\frac{125}{8}+\frac{65}{4}-\frac{15}{2}+1$

$= \frac{-125+150-60+8}{8}$

$= \frac{-27}{8}$

Hence remainder = $\frac{-27}{8}$

Q.2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Ans. Let $p(x) = x^3 - ax^2 + 6x - a$

Let $x - a = 0$

$\Rightarrow \quad x = a$

$\therefore$ Remainder $= p(a)$

$= a^3 - a(a)^2 + 6a - a$

$= a^3 - a^3 + 5a = 5a$

Hence remainder = **$5a$**.

Q.3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Ans. Let $f(x) = 3x^3 + 7x$

Let $7 + 3x = 0$

$\Rightarrow \quad x = -\frac{7}{3}$

$\therefore \quad f\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$

$= -\frac{343}{9}-\frac{49}{3}$

$= \frac{-343-147}{9}$

$= -\frac{490}{9} \neq 0$

Thus remainder $\neq 0$.

Hence $7 + 3x$ is not a factor of $3x^3 + 7x$.

EXERCISE 2.4

Q.1. Determine which of the following polynomials has $(x + 1)$ a factor :

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Ans. (i) $x^3 + x^2 + x + 1$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x + 1 = 0$

$\Rightarrow \quad f(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

Hence by factor theorem, $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$

(ii) $f(x) = x^4 + x^3 + x^2 + x + 1$

Let $x + 1 = 0$

$\Rightarrow x = -1$

$f(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 \neq 0$

Hence by factor theorem, x +1 is a factor of

$x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

$f(x) = x^3 + 3x^2 + 3x + 1$

Let $x + 1 = 0$

$\Rightarrow x = -1$

$f(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0.$

Hence by factor theorem, $x + 1$ is not factor of

$x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}.$

$f(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}.$

Let $x + 1 = 0$

$\Rightarrow x = -1$

$\therefore f(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2} \neq 0$

Hence by factor theorem , $x + 1$ is not factor of

$x^3 - x2 - (2 + \sqrt{2})x + \sqrt{2}.$

Q.2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Ans. (i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

$p(x) = 2x^3 + x^2 - 2x - 1$

$g(x) = x + 1$

Let $g(x) = 0$

$\Rightarrow x + 1 = 1$

$\Rightarrow x = -1$

$\therefore$ Remainder $= p(-1)$

$= 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

Hence $g(x)$ is a factor of $p(x)$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

$p(x) = x^3 + 3x^2 + 3x + 1$

$g(x) = x + 2$

Let $g(x) = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

Now remainder $p(-2)$

$= (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1 = -1 \neq 0$

Hence $g(x)$ is not a factor of $p(x)$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

$p(x) = x^3 - 4x^2 + x + 6$

$g(x) = x - 3$

Let $g(x) = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

No remainder

$= g(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 3 + 6 = 0$

Hence $g(x)$ if a factor of $p(x)$

Q.3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - 2x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Ans. (i) $p(x) = x^2 + x + k$

$g(x) = x - 1$

As g(x) is factor of p(x), therefore, $x - 1$ is a factor of $p(x)$.

$\therefore p(1) = 0$

$(1)^2 + 1 + k = 0$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

As x –1 is a factor of

$p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 0$

Now $p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$= 2 + k + \sqrt{2}$

$\therefore 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -2 - \sqrt{2}.$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

As x – 1 is a factor of p(x).

$\therefore p(1) = 0$

Now $p(1) = k(1)^2 - \sqrt{2}(1) + 1$

$= k - \sqrt{2} + 1$

As $p(1) = 0$

$\therefore \sqrt{k} - 2 + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1.$

(iv) $p(x) = kx^2 - 3x + k$

As $x - 1$ is a factor of p(x)

$\therefore p(1) = 0$

Now p(1) $= k(1)^2 - 3(1) + k$

$= k - 3 + k$

$= 2k - 3$

As $p(1) = 0$

$2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

Q.4. Factorise :

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Ans. (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(3x - 1)$

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 2) - 2(2x - 3)$

$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$

$= 3x^2 - 6x - 2x - 4$

$= 3x(x - 2) - 2(x - 2)$

$= (x - 2)(3x - 2)$

Q.5. Factorise :

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Ans. (i) $x^3 - 2x^2 - x + 2$

Let $p(x) = x^3 - 2x^2 - x + 2$

Now the factors of 2 are + 1, +2 we observe that

$p(1) = (1)^3 - 2(1)^2 - 1 + 2$

$= 1 - 2 - 1 + 2 = 0$

$p(-1) = (-1)^3 - 2(1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2 = 0$

Now $x^3 - 2x^3 - x + 2$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)[x(x+1) - (x+1)]$

$= (x - 1)(x+1)(x+2)$

Ans. (ii) $x^3 - 3x^2 - 9x - 5$

$f(x) = x^3 - 3x^2 - 9x - 5$

Now the factors of 5 are +1, + 5

$f(1) = 1^3 - 3(1)^2 - 9 \times 1 - 5$

$= 1 - 3 - 9 - 5 \neq 0$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

$\therefore x + 1$ is a factor of $f(x)$

$\therefore x^3 - 3x^2 - 9x - 5$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)[x^2 - 5x + x - 5)$

$= (x + 1)[x(x - 5) + (x - 5)$

$= (x + 1)(x+1)(x-1).$

Ans. (iii) $x^3 + 13x^2 + 32x + 20$

$f(x) = x3 + 13x^2 + 32x + 20$

Factors of 20 are +1, + 2, + 4, + 5, + 10, + 20

$f(1) = 1^3 + 13(1) + 32(1) + 20 \neq 0$

$f(-1) = (-1)^3 + 13(-1) + 32(-1) + 20$

$= -1 - 13 - 32 + 20 \neq 0$

$f(2) = (2)^3 + 13(2) + 32(2) + 20 \neq 0$

$f(-2) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$

$= -8 + 52 - 64 + 20 = 0$

$\therefore x + 2$ is a factor of $f(x)$.

$\therefore f(x) = (x + 2)(x^2 + 11x + 10)$

$= (x + 2)[x^2 + 10x + x + 10]$

$= (x + 2)[x(x + 10] + 1(x + 10)]$

$= (x + 2)(x + 10)(x + 1)$

(iv) $2y^3 + y^2 - 2y - 1$

$= y^2(2y + 1) - 1(2y + 1)$

$= (2y +)(y^2 - 1)$

$= (2y + 1)(y + 1)(y - 1)$

EXERCISE 2.5

Q.1. Use suitable identities to find the following products:

(i) $(x + 4)(x + 10)$

(ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

(v) $(3 - 2x)(3 + 2x)$

Ans. (i) $(x + 4)(x + 10)$

(i) Using the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$, we have

$(x + 4)(x + 10) = x^2 + (4 + 10)x + (4)(10)$

$= x^2 + 14x + 40.$

(ii) $(x + 8)(x - 10)$

Again using the identity.

$(x + a)(x + b) = x^2 + (a + b)x + ab$, we have

$(x + 8)(x - 10) = x^2 + [8 + (-10)]x + (8)(-10)$

$= x^2 - 2x - 80.$

(iii) $(3x + 4)(3x - 5)$

$= (3x)^2 + (4 - 5)3x + 4(-5)$

$= 9x^2 - 3x - 9.$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = (y^2)^2-\left(\frac{3}{2}\right)^2$

[Using the identity $(x + y)(x - y) = x^2 - y^2$]

$= y^4-\frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

$= (3)^2 - (2x)^2$

$= 9 - 4x^2$.

[Using the identity $(x + y)(x - y) = x^2 - y^2$]

Q.2. Evaluate the following products without multiplying directly:

(i) 103×107 (ii) 95×96 (iii) 104×96

Ans. (i) 103×107

Using the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3+7) \times 100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

Using the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + [(-5) + (-4)] \times 100 + (-5) \times (-4)$

$= 10000 - 900 + 20$

$= 9120$.

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

[Using the identity $(a + b)(a - b) = a^2 - b^2$]

$= 9984$.

Q.3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$

(iii) $x^2-\frac{y^2}{100}$

Ans. (i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + y^2$

[Using the identify $a^2 + 2ab + b^2 = (a + b)^2$]

$= (3x + y)^2$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

[Using the identity $a^2 - 2ab + b^2 = (a - b)^2$]

$= (2y - 1)^2$

(iii) $x^2-\frac{y^2}{100} = (x)^2-\left(\frac{y}{100}\right)^2$

[Using identity $a^2 - b^2 = (a + b)(a - b)$]

$=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

Q.4. Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Ans. (i) $(x + 2y + 4z)^2$

$= x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

[Using identity (v)]

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

[Using identity (v)]

$= 5x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) +2(3y)(2z) + 2(2z)(-2x)$

[Using identity (v)]

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

[Using identity (v)]

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

[Using identity (v)]

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

$=\left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+1^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)$

$$+2\left(-\frac{1}{2}b\right)(1)+2(1)\left(\frac{1}{4}a\right)$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{ab}{4}-b+\frac{1}{2}a$$

Q.5. Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

Ans. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-2x + y + 2\sqrt{2}z)^2$

Q.6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

Ans. (i) $(2x + 1)^3$

$= (2x)^3 + 1^3 + 3(2x)(1)(2x + 1)$

(Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x + y)$]

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$\

(ii) $(2a - 3b)^3$

$= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

[Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

$$=\left(\frac{3}{2}x\right)^3+1^3+3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$$

[Using identity $(x+y)3 = x^3 + y^3 + 3xy(x+y)$]

$$=\frac{27}{8}x^3+1+\frac{9}{2}x\left(\frac{3}{2}x+1\right)$$

$$=\frac{27}{8}x^3+1+\frac{27}{3}x^2+\frac{9}{2}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $\left(x-\frac{2}{3}y\right)^3$

$$=x^3-\left(\frac{2}{3}y\right)^3-3x\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$$

[Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$$=x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$

$$=x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

$$=x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$$

Q.7. Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Ans. (i) $(99)^3$

$= (100-1)^3 = 100^3 - (1)^3 - 3\times 100 \times 1(100-1)$

[Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

$= (100+2)^3 = 100^3 + 2^3 + 3\times 100\times 2(100+2)$

(Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$]

$= 1000000 + 8 + 600(100+2)$

$= 1000000 + 8 + 60000 + 1200 + 1061208$

(iii) $(998)^3$

$= (1000-2)^3 = 1000^3 - 2^3 - 3\times 1000\times 2(1000-2)$

(Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$]

$= 1000000000 - 8 - 6000(1000-2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

Q.8. Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p.$

Ans. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Using the formula :

$x^3 + y^3 + 3xy(x+y) = (x+y)^3$, we have

$8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + b^3 + 3(2a)(b)(2a+b)$

$= (2a + b)^3$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - b^3 - 3(2a)(b)(2a-b)$

$= (2a-b)^3$.

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$

$= (3 - 3b)^3$.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$.

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3$

Q.9. Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^3)$

Ans. (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

R.H.S. $= (x + y)(x^2 - xy + y^2)$

$= x(x^3 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 =$ L.H.S.

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^3)$

R.H.S. $= (x - y)(x^2 + xy + y^3)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^2$

$= x^3 - y^3 =$ L.H.S.

Q.10. Factorise each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Ans. (i) $27y^3 + 125z^3$

Using the formula

$a^3 + b^3 = (a + b)(a^2 + b^3 - ab)$, we have

$27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

$= (3y + 5z)[(3y)^2 + (5z)^2 - 3y)(5z)]$

$= \mathbf{(3y + 5z)(9y^2 + 25z^2 - 15yz)}$

(ii) $64m^3 - 343n^3$

Using the formula

$a^3 - b^3 = (a - b)(a^2 + b^3 + ab)$, we have $64m^2 - 343n^3$

$= (4m)^3 - (7n)^3$

$= (4m - 7n)[(4m)^2 + (7n)^2 + (4m)(7n)]$

$= \mathbf{(4m - 7n)(16m^2 + 49n^2 + 28\,mn)}$

Q.11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Ans. Using the formula :

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$, we have

$27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + y^3 + z^3 - 3(3)(y)(z)$

$= (3x + y + z)$

$[(3x)^2 + y^2 + z^2 - (3x)(y) - y(z) - z(3x)]$

$= (3x + y + z)(9z^2 + y^2 + z^2 - 3xy - yz - 3xz)$

Q.12. Verify that :

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (x - x)^2]$

Ans. We know that :

L.H.S. $= x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= \frac{1}{2}(x + y + z)$

$[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2}(x + y + z)[x^2 + 2y^3 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2zx)$

$= \frac{1}{2}[x + y + z][(x - y)^2 + (y - z)^2 + (z + x)^2]$

= R.H.S.

Q.13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Ans. We know that

$x^3 + y + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 0 \times (x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^2 - 3xyz = 0$

$[x + y + z = 0]$ (given)

Hence $x^3 + y^3 + z^3 = 3xyz$.

Q.14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Ans. (i) $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$

$y = 7$

$z = 5$

Now, $x + y + z = -12 + 7 + 5 = 0$

$\therefore\ x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = 3 \times (-12)(7)(5)$

$= \mathbf{-1260}$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28$

$y = -15$

$z = -13$

Now, $x + y + z = 28 - 15 - 13 = 0$

$\therefore \quad x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow \quad (28)^3 + (-15)^3 + (-13)^3$

$= 3(28)(-15)(-23)$

$= 84 - 15 - 13$

$= 1260 - 13$

$= \mathbf{16380}$

Q.15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

Ans. (i) Area : $25a^2 - 35a + 12$

$= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a - 4) - 3(5a - 4)$

$= (5a - 4)(5a - 3)$

Hence length $= 5a - 4$

and Breadth $= 5a - 3$

(ii) Area : $35y^2 + 13y - 12$

$= 35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (7y - 3)(5y + 4)$

Hence length $= 7y - 3$

and Breadth $= 5y + 4$

Q.16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 - 12x$

(ii) Volume : $12ky^2 + 8ky - 20k$

Ans. (i) Volume $= 3x^2 - 12x$

$= 3x(x - 4)$

Hence length $= 3$

breadth $= x$

and height $= x - 4$

(ii) Volume $= 12ky^2 + 8ky - 20k$

$= 2k(6y^2 + 4y - 10)$

Hence length $= 2$

breadth $= k$

and height $= 6y^2 + 4y - 10.$

Additional Questions

Q.1. Classify the following polanomials in one variable, two variables etc.

(i) $x^2 + x + 1$

(ii) $y^3 - 5y$

(iii) $xy + yz + zx$

(iv) $x^2 - 2xy + y^2 + 1$

Ans. (i) $x^2 + x + 1$ is a polynomial in one variable.

(ii) $x^3 - 5y$ is a polynomial in one variable.

(iii) $xy + yz + zx$ contains 3 variable.

So, it is a polynomial in three variables.

(iv) $x - 2xy + y^2 + 1$ is a polynomial in two variables.

Q.2. For the polynomial

$$\mathbf{\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6}, \textbf{ write}$$

(i) the degree of the polynomial

(ii) the coefficient of x^3

(iii) the coefficient of x^6

(iv) the constant term

Ans. We have

$$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$$

$$= \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$

(i) Degree of the polynomial = 6

(ii) The coefficient of $x^3 = \frac{1}{5}$

(iii) The coefficient of $x^6 = -1$

(iv) The constant term of $\frac{1}{5}$.

Q.3. If $a + b + c = 17$ and $ab + bc + ca = 20$, find th value of $a^2 + b^2 + c^2$.

Ans. We have

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow \quad (7)^2 = a^2 + b^2 + c^2 + 2 \times 20$

$\Rightarrow \quad a^2 + b^2 + c^2 = 49 - 40 = 9$

Q.4. Evaluate $(104)^3$ using suitable identity.

Ans. $(104)^3 = (100 + 4)^3$

$= (100)^3 + (4)^3 + 3(100)(4)(100+4)$

[Using identity $(a + b)^3 = a^3 + b^3 + 3ab(a+b)$]

$= 1000000 + 64 + 1200(100 + 4)$

$= 1000000 + 64 + 1200 \times 100 + 1200 \times 4$

$= 1000000 + 64 + 120000 + 4800$

$= 1124864.$

Q.5. Factorise : $a - b - a^3 + b^3$.

Ans. $a - b - a^3 + b^3$

$= (a - b) - (a^3 - b^3)$

$= (a - b) - (a - b)(a^2 + ab + b^2)$

$= (a - b)(1 - a^2 - ab - b^2)$

$= (a - b)(1 - a^2 - ab - b^2)$.

Q.6. Using remainder theorm, find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $\left(x - \frac{1}{2}\right)$.

Ans. Let $P(x) = x^3 + 3x^2 + 3x + 1$

$$x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$$

$$\therefore \text{Remainder} = P\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3 \times \left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + \frac{1}{1}$$

$$= \frac{1 + 6 + 12 + 8}{8} = \frac{27}{8}$$

Q.7. Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.

Ans. $x^3 - 8y^3 - 36xy - 216$

$= (2y + 6)^3 - 8y^3 - 36(2y + 6)y - 216$

$= (2y)^3 + (6)^3 + 3 \times 2y \times 6(2y + 6) - 8y^3 - 72y^2 - 216y - 216$

$= 8y^3 + 216 + 72y^2 + 216y - 8y^3 - 72y^2 - 216y - 216 = 0$

Q.8. Which of the following expressions are polynomials ? Justify your answer.

(i) 8 (ii) $\frac{1}{x+1}$

(iii) $\frac{1}{x+1}$

Ans. (i) Yes, $8 = 8x^0$

It is a polynomial because it can be written as non-negative integral power of x.

(ii) No. It is not a polynomial bcause it cannot be written as non-negative integral power of x.

(iii) Yes, it is a polynomial because it is written as a non-negative integral power of a.

Q.9. Write whether the following statements are True or False. Justify your answer.

The degree of the sum of two polynomials each of degree 5 is always 5.

Ans. False. because the sum of any two polynomials of same degree is not always same degree.

e.g. Let $f(x) = x^5 + 1$

and $g(x) = x^5 + 2x + 3$

$\therefore$ $f(x) + g(x) = x^5 + 1 - x^5 + 2x + 3$

$= 2x + 4$.

Q.10. If x and y are two positive real numbers such that $x > 3y$, $x^2 + 9y^2 = 369$ and $xy = 60$, find the value of $x - 3y$.

Ans. We have :

$x^2 + 9y^2 = 369$ (Given)

$xy = 60$ (Given)

$\therefore$ $(x - 3y)^2 = x^2 + (3y)^2 - 2.x.3y$

$(x - 3y)^2 = x^2 + 9y^2 - 6xy$

$= 369 - 6 \times 60 = 369 - 360$

$= 9$

$\Rightarrow$ $x - 3y = 3$

Multiple Choice Questions

Q.1. The remainder when $x^{31} + 31$ is divided by $x + 1$ is :

(a) 30 (b) 31

(c) – 1 (d) 0

Ans. (a)

Q.2. Degree of the polynomial

$p(x) = 4x^4 + 2x^2 + x^5 + 2x + 7$ is

(a) 7 (b) 4

(c) 5 (d) 3

Ans. (c)

Q.3. Select the correct statement from the following:

(a) Degree of a zero polynomial is 0.

(b) Degree of a zero polynomial is not defined.

(c) Degree of a constant polynomial is not defined.

(d) Zero of a zero polynomial is not defined

Ans. (b)

Q.4. If $x - 1$ is a factor of $p(x) = x^2 + x - k$ then value of k is :

(a) 1 (b) –1
(c) 0 (d) 2

Ans. (d)

Q.5. Zero of polynomial p(x) = 2x + 5 is :

(a) $\frac{-2}{5}$ (b) $\frac{-5}{2}$

(c) $\frac{2}{5}$ (d) $\frac{5}{2}$

Ans. (b)

Q.6. If a + b + c = 0 then $a^3 + b^3 + c^3$ is equal to :

(a) 0 (b) abc
(c) 2abc (d) 3 abc

Ans. (d)

Q.7. The remainder obtained when the polynomial p(x) is divided by (b – ax) is :

(a) $p\left(\frac{-b}{a}\right)$ (b) $p\left(\frac{-a}{b}\right)$

(c) $p\left(\frac{a}{b}\right)$ (d) $p\left(\frac{b}{a}\right)$

Ans. (d)

Q.8. Product of $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)$ is :

(a) $x^4+\frac{1}{x^4}$ (b) $x^3+\frac{1}{x^3}-2$

(c) $x^4-\frac{1}{x^4}$ (d) $x^2+\frac{1}{x^2}+2$

Ans. (c)

Q.9. Find $p\left(\frac{1}{3}\right)$ for $p(t) = t^2 - t + 2$:

(a) $\frac{22}{9}$ (b) $\frac{14}{9}$

(c) $\frac{16}{9}$ (d) $\frac{15}{9}$

Ans. (c)

Q.10. The value of $p\left(\frac{1}{2}\right)$ for $p(x) = x^4 - x^2 + x$ is :

(a) $\frac{7}{16}$ (b) $\frac{5}{16}$

(c) $\frac{3}{16}$ (d) $\frac{1}{16}$

Ans. (b)