Chapter 1

Number Systems

In the Chapter

- **Natural Numbers (N) :** The counting numbers 1, 2, 3, ... are called natural numbers.
- **Whole Numbers (W) :** If we add 0 (Zero) in the set of natural numbers it is called whole numbers.
  i.e., W = (0, 1, 2, 3, ...)
- **Integers (Z) :** The collection of natural numbers, zero and their negatives are known as integers, It is denoted by Z.
  Z = (0, + 1, + 2, + 3, ...)
  Here, –1, –2, –3, ... are known as negative integers and 1, 2, 3, ... are known as positive or non-negative integers.
- **Rational Numbers (R) :** Number of the form $\frac{p}{q}$, q ≠ 0; p and q ∈ Z are known as rational numbers. It is denoted by Q.

$$Q = \left\{\frac{p}{q}, q \neq 0; p, q \in Z\right\}$$

- **Irrational Numbers :** A number S is called irrational if it cannot be written in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0.
  As the rational numbers are infinite, so are the irrational numbers are also infinite.

  Examples : (i) $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, \frac{1}{2+\sqrt{3}}$

  Also s = 0.20200200020000 ......
  Note : When we use the symbol √, we assume that it is positive square root of a number. So, √9 = 3, though both 3 and –3 are square roots of 9.
  (ii) π = 3.142 ... = 3.14152....
  Here π is defined as the ratio of the circumference of any circle to its diameter and it is an irrational number.
- **Decimal Representation of a Rational Number :**
  Rational Numbers : A rational number is either a terminating decimal or a non-terminating but recurring (repeated) decimal.
  In other words, a terminating decimal or a non-terminating but recurring decimal is a rational number.

Recurring decimals are also expressed as below :

(i) $0.33 \ldots = 0.\overline{3}$ (ii) $0.656565 \ldots = 0.\overline{65}$

(iii) $0.12341234\ldots = 0.\overline{1234}$ (iv) $0.8121212\ldots = 0.8\overline{12}$

- The decimal expansion of an irrational number is non-termiating, non recurring. Moreover, a number whose decimal expansion is non-terminating, non-recurring is irrational.
- **Real Numbers :** The set of rational numbers and irrational numbers form a set of real numbers.
- **Properties of Real Numbers :** Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.
- **Operation on real numbers :** Real numbers (rational or irrational) can be added, subtracted, multiplied, divided and we can take square roots or nth root of real numbers.
- There is a unique real number corresponding to every point on the number line. Moreover corresponding to each real number, there is a unique point on the number line.
- Rationalising the denominator : We know that it is easy to handle a number if the denominator is a rational number. We generally remove an irrational from the denominator by certain methods which are explained in the examples below :

i.e. $$\frac{1}{\sqrt{a}+b}=\frac{1}{\sqrt{a}+b}\times\frac{\sqrt{a}-b}{\sqrt{a}-b}=\frac{\sqrt{a}-b}{(\sqrt{a})^2-b^2}=\frac{\sqrt{a}-b}{a-b}$$

- Laws of exponents for real numbers : Let a > 0 be a real number and n be a positive integer. Then

$$n\sqrt{a} = b, \text{ if } b^n = a \text{ and } b > 0.$$

The symbol $\sqrt{}$ is called the radical sign. Also in the language of exponents we define.

$$n\sqrt{a} = a^{1/n}$$

- We call the process of visualisation of representation of numbers on the number line, through a magnifying glass, as the Process of successive magnification.
- Thus, we can say that every real number is represented by a unique point on the number line. Further, every point on the number line represents one and only one real number.
- The sum or difference of a rational number and an irrational number is irrational.
- The product or quotient of a non-zero rational number with an irrational number is irrational.
- If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.
- When the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 1.1

Q.1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Ans. Yes, zero is a rational number.

It can be written in the form $\frac{p}{q}$.

Where $p = 0$, $q = 1, 2, \ldots\ldots$

$\therefore \quad 0=\frac{0}{1}=\frac{0}{2}\ldots\ldots etc.$

where p and q are integers and $q \neq 0$.

Q.2. Find six rational numbers between 3 and 4.

Ans. We have to find six rational numbers between 3 and 4.

So, denominator should be made equal to 6 + 1 = 7.

$\therefore \quad 3 = \frac{3\times 7}{1\times 7} = \frac{21}{7}$

and $\quad 4 = \frac{4\times 7}{1\times 7} = \frac{28}{7}$

$\therefore$ The six numbers are :

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$$

Alternative method : We have to find 6 rational numbers between 3 and 4.

The numbers, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9 are between 3 and 4.

Q.3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans. We have to find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

We know that

$$\frac{3}{5} = 0.6 = 0.60$$

and $\quad \frac{4}{5} = 0.8 = 0.80$

Hence five required rational numbers are

0.61, 0.62, 0.63, 0.71, 0.72

Alternative method :

$$\frac{3}{5} = \frac{3\times 10}{5\times 10} = \frac{30}{50}$$

and $\quad \frac{4}{5} = \frac{4\times 10}{5\times 10} = \frac{40}{50}$

$\therefore$ Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$. **Ans.**

Q.4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Ans. (i) **True,** every natural number is a whole number.

(ii) **False,** every integer is not a whole number, because negative integers are not whole numbers.

(iii) **False,** every rational number is not a whole number, because fractional rational numbers (like 5/8) are not whole numbers.

EXERCISE 1.2

Q.1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Ans. (i) True, since real numbers constitute the set of rational and irrational numbers.

(ii) False, –3 is a real number, but it is not the square root of any natural number.

(iii) False, 3 is a real number but 3 is not an irrational number.

Q. 2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Ans. No. Since $\sqrt{9}$ = 3 is a rational number and irrational numbers.

Q.3. Show how $\sqrt{5}$ can be represented on the number line.

Ans. Draw a number line as shown in Fig. 1.1.

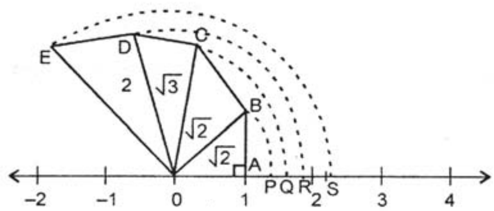


Fig. 1.1.

Let 1 be named as A.

Draw AB = 1 unit.

Now $\quad OB = \sqrt{1^2 + 1^2} = \sqrt{2}$

With O as centre OB = $\sqrt{2}$ as radius, draw a point P on the number line which represents $\sqrt{2}$. Again at OB, draw BC $\perp$ ON and take OB = 1 unit.

Now $\quad OC = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{3}$

With O as centre and OC = $\sqrt{3}$ as radius, mark a point Q on the number line which denotes $\sqrt{3}$.

In a similar way, we get OD = 2, OE = $\sqrt{5}$.

Thus $\sqrt{5}$ can be represented on the number line.

Q.4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length (see Fig. 1.2). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points $P_2, P_3, \ldots, P_n, \ldots$, and joined them to create a beautiful spiral depicting $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ...

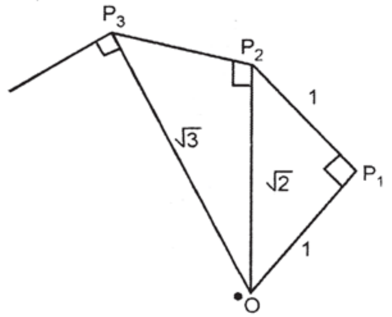


Fig. 1.2 Constructing square root spiral.

Sol. Do yourself Activity 1.

EXERCISE 1.3

Q.1. Write the following in decimal form and / say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Ans. (i) $\frac{36}{100} = 0.36$

It is a terminating decimal.

(ii) $\frac{1}{11}$

11) 1.00 (0.0909
99
100
99
1

$\therefore \frac{1}{11} = 0.0909\ldots = 0.\overline{09}$

It is a non-terminating repeating decimal.

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

It is a terminating decimal.

(iv) $\frac{3}{13}$

13) 3.0 (0.230769
26
40
39
100
91
90
78
120
117
3

$\therefore \frac{3}{13} = 0.\overline{230769}$

It is a non-terminating repeating decimal.

(v) $\frac{2}{11}$

11) 2.0 (0.18
11
90
88
2

$\therefore \quad \frac{2}{11} = 0.18$

It is a non-terminating repeating decimal.

(iv) $\frac{329}{400}$

```
400 ) 329.000 ( 0.8225
      3200
       900
       800
      1000
       800
      2000
      2000
         0
```

$\therefore \quad \frac{329}{400} = 0.8225$

It is a terminating decimal.

Q.2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[**Hint :** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Ans.

```
7 ) 1.0000 ( 0.142857
     7
     30
     28
      20
      14
       60
       56
        40
        35
         50
         49
          1
```

$\frac{1}{7} = 0.142857.....$

To find $\frac{2}{7}$, find the remainder 2 and then respective quotients are 2, 8, 5, 7, 1, 4.

Now $\quad \frac{2}{7} = 0.285714......$

Similarly $\frac{3}{7} = 0.\overline{428571}$

$\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$

and $\quad \frac{6}{7} = 0.\overline{857142}$

Q.3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Ans. (i) Let $x = 0.6 \Rightarrow x = 0.666\ ...$

Since one digit is repeating,

Therefore, we multiply both sides by 10.

$\therefore \quad 10x = 6.66..$

$10x = 6 + 0.666$

$10x = 6 + x$

$9x = 6$

$\Rightarrow \quad x = \frac{6}{9} = \frac{2}{3}$ **Ans.**

(ii) Let $x = 0.4\overline{7}$

$\therefore \quad x = 0.4777\ ...$

Since one digit is repeating,

$\therefore$ we multiply both sides by 10.

$\therefore \quad 10x = 4.777...$

$10x = 4.3 + 0.477\ ....$

$10x = 4.3 + x$

$9x = 4.3$

$\Rightarrow \quad x = \frac{43}{90}$ **Ans.**

(iii) Let $x = 0.001$

$\therefore \quad x = 0.001001...$

$\therefore \quad 1000x = 1.001001\ ...$

$1000x = 1 + 0.001001...$

$1000x = 1 + x$

$999x = 1$

$\Rightarrow \quad x = \frac{1}{999}$ **Ans.**

Q.4. Express 0.99999 .... in the form *p/q*. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Ans. Let

$$\begin{aligned} x &= 0.9999\ldots \\ 10x &= 9.999\ldots \\ 10x &= 9+0.999\ldots \\ 10x &= 9+x \\ 9x &= 9 \\ \Rightarrow \quad x &= 1 \end{aligned}$$

We see that 0.999... goes on forever. So, there in no gap between 1 and 0.999... and hence they are equal.

Q.5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Ans. By actual division, we have,

17) 1.00000 (0.0588235294117647

85
150
136
140
136
40
34
60
51
90
85
50
34
160
153
70
68
20
17
30
17
130
119
110
102
80
68
120
119
1

$$\therefore \quad \frac{1}{17} = 0.0588235294117647$$

When the quotient is 17. Then the maximum number of digits are 16.

Q.6. Look at several examples of rational numbers in the form *p/q* ($q \neq 0$), where *p* and *q* are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property *q* must satisfy?

Ans. Consider the examples

$$\frac{5}{2}, \frac{17}{5}, \frac{108}{25}, \frac{39}{100}$$

We observe that if the quotient of a rational number is either 2 or 5 or their multiples, then the decimal number is a terminatiung decimal. Hence *q* must be a multiple of 2 or 5 or both or their multiple.

Q.7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Ans. Such non-terminating non-recurring decimals are

(i) 0.202002000... (ii) 0.121121112...
(iii) 0.01020304.....

Q.8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans. $\frac{5}{7} = 0.714285$

and $\frac{9}{11} = 0.81$

We know that irrational numbers are non-terminating non-recurring. Therefore three such numbers are :

(i) 0.7207200...
(iii) 0.747576....
(iii) 0.808008000.....

Q.9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478

(v) 1.101001000100001 ...

Ans. (i) $\sqrt{23}$. It is an irrational number.

(ii) $\sqrt{225} = 15$. It is a rational number.

(iii) 0.3796. It is a terminating decimal, so it is a rational number.

(iv) $7.478478 = 7.\overline{478}$

It is a non-terminating recurring decimal, so it is a rational number.

(v) 1.101001000100001 ...

It is a non-terminating non-recurring decimal, so it is an irrational number.

EXERCISE 1.4

Q.1. Visualise 3.765 on the number line, using successive magnification.

Ans. We will proceed by successive magnification process. We know that 3.765 lies between 3 and 4 i.e., in the interval [3,4] and have a rough idea where it is located on the number line. To get a more accurate estimation of the representation, we divide the interval [3,4] into 10 equal parts and look at [3.7, 3.8] though a magnifying glass and realize that 3.765 lies between 3.7 and 3.8 i.e., in the interval p[3.7, 3.9]. Now we may imagine that each of the new interval [3, 3.1], [3.1, 3.2], [3.2, 3.3] ... [3.9,4] has been sub-divided into 10 equal parts. As before, we can now visualize through the magnifying glass that 3.765 lies in the interval [3.76, 3.77].

Hence, we observe that it is possible by sufficient successive magnification to visualize the position of a real number with a terminating decimal expansion on the number lines.

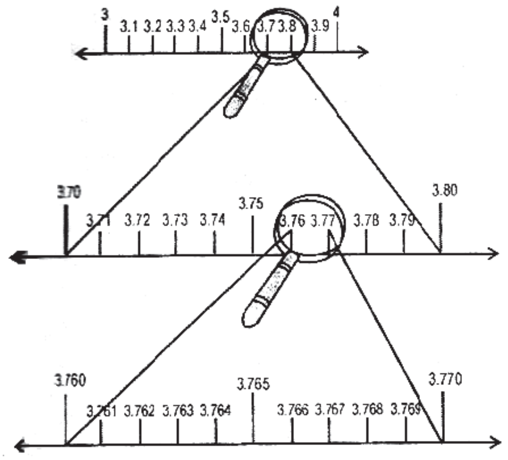


Fig. 1.3.

Alternatively :

Step 1. The given number lies between 3 and 4.

Step 2. Magnify the interval between 3 and 4 divided it into 10 equal parts.

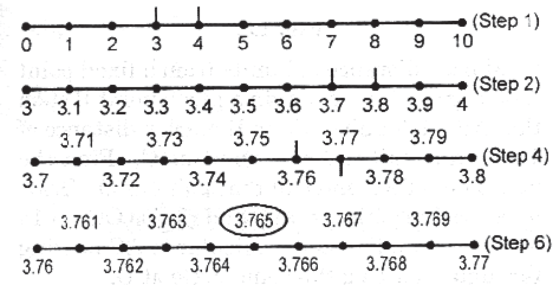
Step 3. The given number lies between 3.7 and 3.8.

Step 4. : Divide the interval 3.7 and 3.8 into ten equal parts and magnify it.

Step 5 : The given number lies between 3.76 and 3.77.

Step 6 : Magnify the interval between 3.76 and 3.77 and divide it into ten equal parts.

Step 7 : 3.675 is the fifth division in the magnification.



Q.2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Ans. Here also we proceed by sucessive magnification and successively decrease the lengths of the intervals in which $4.\overline{26}$ is located. We see that $4.\overline{26}$ is located in the interval [5, 6] of length 1. We now locate $4.\overline{26}$ in the interval [4.2, 4.3] of length 0.1. To get a more accurate visualisation of the representation, we divide even interval into 10 equal parts and use a magnifying glass to visualize that $4.\overline{26}$ lie in the interval [4.26, 4.27] of length 0.0 visualized $4.\overline{26}$ in an interval of length 0.001, we again divide each of the new interval into 10 equal parts and visualize the representation 4.26 in the interval [4.262, 4.263].

Steps at a glance :

Step 1. On the number line the given number 4.26 lies between 4 and 5. (For four decimal places number is 4.2626.)

Step 2. Magnify the interval between 4 and 5 and divide it into 10 equal parts.

Step 3. The given number 4.2626 lies between 4.2 and 4.3.

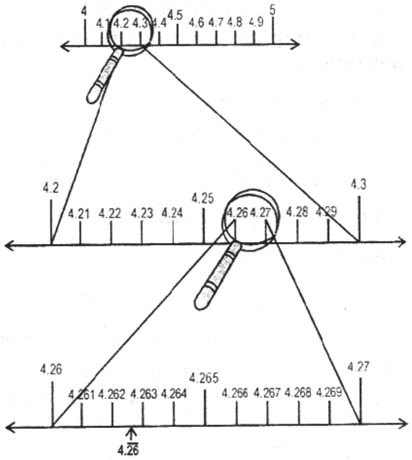


Fig. 1.4.

Step 4. Magnify the interval between 4.2 and 4.3 and divide it into ten equal parts.

Step 5. The given number falls between 4.26 and 4.27

Step 6. Magnify the interval between 4.26 and 4.27 and divide it into ten equal parts.

Step 7. The given number lies between 4.262 and 4.263.

Step 8. Magnify the interval between 4.262 and 4.263 and divide it into ten equal parts.

Step 9. The given number is the sixth division of the given interval.

EXERCISE 1.5

Q.1. Classify the following numbers as rational or irrational :

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\dfrac{1}{\sqrt{2}}$ (v) 2π.

Ans. (i) $2-\sqrt{5}$. It is an **irrational number.**

(ii) $(3+\sqrt{23})-\sqrt{23}$. It is a **rational number.**

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$. It is a **rational number.**

(iv) $\dfrac{1}{\sqrt{2}}$. It is an **irrational number.**

(v) 2π. It is an **irrational number.**

Q.2. Simply each of the following expressions :

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3+\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Ans. (i) $(3+\sqrt{3})(2+\sqrt{2})$

$= 3(2+\sqrt{2}) + \sqrt{3}(2+\sqrt{2})$

$= 6 + 3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3+\sqrt{3})$

Ans. $(3+\sqrt{3})(3+\sqrt{3}) = (3)^2-(\sqrt{3})^2$

[$\therefore$ (a+b) (a–b) = a^2b^2]

$= 9-3 = 6.$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Ans. $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+(\sqrt{2})^2+2.\sqrt{5}.\sqrt{2}$

[$(a + b)^2 = a^2 + b^2 + 2ab$)

$= 5+2+2\sqrt{10}$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= (\sqrt{5})^2-(\sqrt{2})^2$

[$\therefore$ (a + b) (a – b) = $a^2 - b^2$]

$= 5-2 = 3.$

Q.3. Recall, π is defined as the ratio of the circumference (say *c*) of a circle to its diameter (say *d*). That is, $\pi = \dfrac{c}{d}$ .This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Ans. Activity $\dfrac{c}{d} = \dfrac{22}{7}$ which is an approximate value of π.

Q.4. Represent √9.3 on the number line.

Ans. Firstly we draw AB = 9.3 units. Now, from

B, mark a distance of 1 unit. Let this point be C. Let O be the mid-point of AC. Now, draw a semi-circle with centre O and radius OA. Let us draw a line perpendicular to AC passing through point B and intersecting the semi-circle at point D.

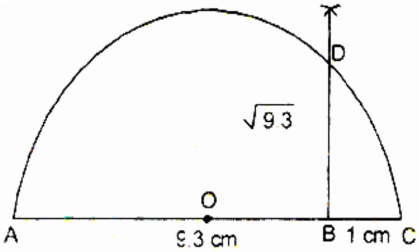


Fig. 1.5.

$\therefore$ The distance BD = √9.3.

Draw an arc with centre B and radius BD, which intersects the number line at point E, then the point E represents √9.3.

Q.5. Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ **(ii)** $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ **(iv)** $\frac{1}{\sqrt{7}-2}$

Ans. (i) $\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{7}$ (Multiplying and dividing by $\sqrt{7}$)

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} \Rightarrow \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$ (Multiplying and dividing by √7 + √6)

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} =$

$\frac{\sqrt{5}-\sqrt{2}}{5-2}=\frac{\sqrt{5}-\sqrt{2}}{3}$ (Multiplying and dividing by $\sqrt{5}-\sqrt{2}$)

(iv) $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{(\sqrt{7})^2-2^2} =$

$\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$ (Multiplying and dividing by √ 7 + 2)

EXERCISE 1.6

Q.1. Find

(i) $64^{\frac{1}{2}}$ **(ii)** $32^{\frac{1}{5}}$ **(iii)** $125^{\frac{1}{3}}$

Ans. (i) $64^{\frac{1}{2}} = (8\times8)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8$ [$\therefore$ $(a^m)^n = a^{mn}$]

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{\left(5\times\frac{1}{5}\right)} = 2^1 = 2.$

(iii) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{\left(3\times\frac{1}{3}\right)} = 5^1 = 5.$

Q.2. Find

(i) $9^{\frac{1}{2}}$ **(ii)** $32^{\frac{2}{5}}$ **(iii)** $16^{\frac{3}{4}}$

(iv) $(125)^{\frac{1}{3}}$

Ans. (i) $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}} = 3^2 = 3.$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4.$

(iii) $16^{\frac{3}{4}} = (2^4)^{\frac{1}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 8.$

(iv) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\left(-\frac{1}{3}\right)} = 3^{-1} = \frac{1}{3}.$

Q.3. Simplify :

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ **(ii)** $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ **(iv)** $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Ans.(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = 3^{\frac{1}{3}\times 7} = 3^{\frac{7}{3}}$

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$

Additional Questions

Q.1. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number ? Give an example in support of your answer.

Ans. Yes, Let $x = 21$, $y = \sqrt{2}$ be a rational number.

Now, $x + y = 21 + \sqrt{2} = 21 + 1.4142 = 22.4142..$

which is non-terminating non-recurring. Hence $x + y$ is irrational.

Q.2. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Ans. No. $0 \times \sqrt{2} = 0$ which is not irrational.

Q.3. State whether the following statements are true or false? Justify your answer.

(i) $\dfrac{\sqrt{2}}{3}$ is a rational number.

(ii) There are infinitely many integers between any two integers.

(iii) Number of rational numbers between 15 and 18 is finite.

(iv) There are numbers which cannot be written in the form $\dfrac{p}{q}$, $q \neq 0$, p, q both are integers.

(v) The square of an irrational number is always rational.

(vi) $\dfrac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as 12 and 3 are not integers.

(vii) $\dfrac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\dfrac{p}{q}$, $q \neq 0$, p, q and so it is a rational number.

Ans. (i) False. Although $\dfrac{\sqrt{2}}{3}$ is of the form $\dfrac{p}{q}$ but here p, i.e., $\sqrt{2}$ is not an integer.

(ii) False. Between 2 and 3, there is no integer.

(iii) False, because between any two rational numbers we can find infinitely many rational numbers.

(iv) True. $\dfrac{\sqrt{2}}{\sqrt{3}}$ is of the form $\dfrac{p}{q}$ but p and q here are not integers.

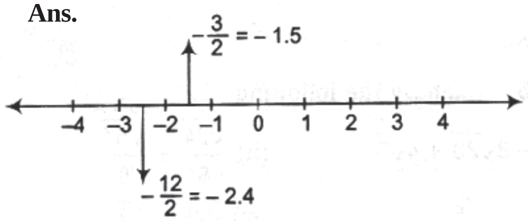
(v) False, as $(\sqrt[4]{2})^2 = \sqrt{4}$ = 2 which is not a rational number.

(vi) False, because $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{4}$ = 2 which is a rational number.

(vii) False, because $\dfrac{\sqrt{15}}{\sqrt{3}} = \sqrt{5} = \dfrac{\sqrt{5}}{1}$ which is p, i.e., $\sqrt{5}$ is not an integer.

Q.3. Represent the following numbers on the number line $-\dfrac{3}{2}$ and $-\dfrac{12}{5}$.

Ans.



Q.4. Express the following in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.888 .... **(ii) 5.2**

Ans. (i) Let $x = 0.888....$ ... (i)

On multiplying Eq. (i) by 10, we get

$10x = 8.888....$

On subtracting Eq.(ii) from Eq. (i), we get ...(ii)

$\Rightarrow \quad 10x - x = (8.88...) - (0.888...)$

$\Rightarrow \quad 9x = 8$

$\Rightarrow \quad x = \frac{8}{9}$

(iii) Let $x = 5.222...$ ... (iii)

On multiplying Eq. (iii) by 10. we get

$10x = 52.222$

On subtracting Eq. (iv) from Eq. (iii), we get

$\Rightarrow \quad 10x - x = (52.222..) - (5.222...)$

$\Rightarrow \quad 9x = 47$

$\Rightarrow \quad x = \frac{47}{9}$

Q.5. Show that 0.142857142857... = $\frac{1}{7}$

Ans. Let $x = 0.142857142857$ .... (i)

On multiplying Eq. (i) by 1000000, we get

$1000000\,x = 142857.142857....$

On subtracting Eq. (ii) from Eq. (i), we get

$1000000\,x - x = (142857.142857..) - (0.142857)$

$999999\,x = 142857$

$\therefore \quad x = \frac{142857}{999999}$

$x = \frac{1}{7}$

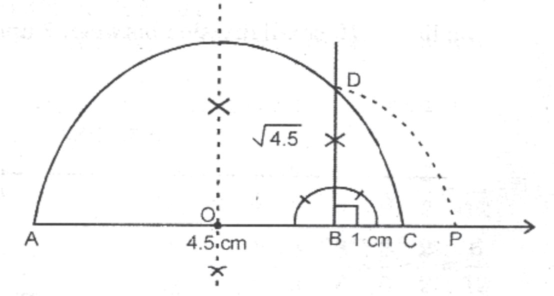
Hence Proved.

Q.6. Represent geometrically the following numbers on the number line :

(i) $\sqrt{4.5}$ (ii) $\sqrt{5.6}$

(iii) $\sqrt{8.1}$ (iv) $\sqrt{2.3}$

Ans. (i) Representation of $\sqrt{4.5}$



Mark a distance 4.5 units from a fixed point A on a given line to obtain a given point B such that AB = 4.5 units. From B mark a distance of 1 unit and call the new point as C. Find the mid-point of AC and call that point as O. Draw a semi-circle with centre O and radius OC = 2.75 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D.

$$\frac{5.5}{2} = 2.75$$

Then BD = BP = $\sqrt{4.5}$

(ii) Representation of $\sqrt{5.6}$ on the number line:

Steps of Construction :

(i) Draw a line and mark a point Q on it.

(ii) Mark a point O on the line drawn in step (i) such that QO = 5.6 cm.

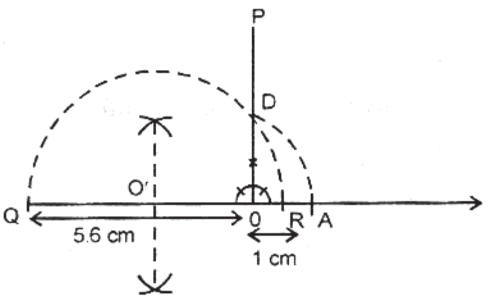
(iii) Mark a point R on QO produced such that OR = 1 unit.

(iv) Draw the perpendicular bisector of QA to obtain the mid-point of it. Let the mid-point be O'.

(v) Taking O' as the centre and O'R = O'Q as radius draw a semi-circle. Also, draw a line passing through O perpendicular to O'R. Suppose it cuts the semi-circle at D.

(vi) Taking O as centre and OD as radius draw an arc cutting O'R produced at A.

Hence, 'A' represents $\sqrt{5.6}$ on the number line.



(iii) $\sqrt{8.1}$ (Proceed same as (i) or (ii) above)

(iv) $\sqrt{2.3}$ (Proceed same as (i) or (ii) above)

Q.7. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans. $\frac{3}{4} = \frac{3 \times 10}{5 \times 10} = \frac{30}{50}$

$$\frac{4}{5}=\frac{4\times10}{5\times10}=\frac{40}{50}$$

$\therefore$ Five rational numbers are :

$$\frac{31}{50},\frac{32}{50},\frac{33}{50},\frac{34}{50},\frac{35}{50}$$

Q.8. Simplify : $(3+\sqrt{3})(2+\sqrt{2})^2$

Ans. $(3+\sqrt{3})(2+\sqrt{2})^2$

$= (3+\sqrt{3})[(2)^2+2.2(\sqrt{2})+(\sqrt{2})^2]$

$= (3+\sqrt{3})[4+4\sqrt{2}+2)$

$= (3+\sqrt{3})(6+4\sqrt{2})$

$= 18+12\sqrt{2}+6\sqrt{3}+4\sqrt{6}.$

Q.9. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponential form of x.

Ans. $\sqrt[4]{\sqrt[3]{x^2}} = \sqrt[4]{(x^2)^{\frac{1}{3}}} = \sqrt[4]{x^{\frac{2}{3}}}$

$$=\left(x^{\frac{2}{3}}\right)^{\frac{1}{4}} = x^{\frac{2}{3}\times\frac{1}{4}} = x^{\frac{1}{6}}$$

Q.10. Evaluate $\dfrac{36^{\frac{7}{2}}-36^{\frac{-9}{2}}}{(36)^{\frac{-5}{2}}}$.

Ans. We have

$$\frac{36^{\frac{7}{2}}-36^{\frac{-9}{2}}}{(36)^{\frac{-5}{2}}} = \frac{(6^2)^{\frac{7}{2}}-(6^2)^{\frac{-9}{2}}}{(6^2)^{\frac{-5}{2}}}$$

$$=\frac{6^7-6^{-9}}{6^{-5}} = 6^5(6^7-6^{-9})$$

$$= 6^{5+7} - 6^{5+(-9)} \left[\because \frac{1}{x^{-m}} = x^m\right]$$

$$= 6^{12} - 6^{-4} = 6^{12} - \frac{1}{6^4}$$

Multiple Choice Questions

Q.1. Which of the following is a rational number.
- (a) $\sqrt{4}$
- (b) π
- (c) 0.101001000100001..
- (d) 0.853853853

Ans. (a) and (d) both

Q.2. π is a /an :
- (a) Natural number
- (b) Integer
- (c) Rational number
- (d) Irrational number

Ans. (d) π is an irrational number.

Q.3. A number is an irrational if and only if its decimal representation i s
- (a) non-terminating
- (b) non-terminating and repeating
- (c) non-terminating and non-repeating
- (d) terminating

Ans. (c) non-terminating and non-repeating

Q.4. Every rational number is
- (a) a natural number
- (b) an integer
- (c) a real number
- (d) a whole number

Ans. (c) Since, real numbers are the combination of rational and irrational numbers.

Q.5. Between two rational numbers
- (a) there is no rational number
- (b) there is exactly one rational number.
- (c) there are infinitely many rational numbers
- (d) there are only rational numbes and no irrational numbers.

Ans. (c) e.g. $\frac{3}{5}$ and $\frac{4}{5}$ are two rational numbers, then $\frac{31}{50},\frac{32}{50},\frac{33}{50},\frac{34}{50},\frac{35}{50}$ .. are infinite rational numbers between them.

Q.6. Decimal representation of a rational number cannot be
- (a) terminating

(b) non-terminating
(c) non-terminating repeating
(d) non-terminating non-repeating.

Ans. (d) The decimal expansion of rational number is either terminating or non-terminating recurring.

Q.7. The produce of any two irrational number is

(a) always an irrational number
(b) always an rational number
(c) always an integer
(d) sometimes rational, sometimes irrational

Ans. (d) e.g. $\sqrt{2} \times \sqrt{2} = 2$ (rational) and $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (irrational)

Q.8. The decimal expansion of the number $\sqrt{2}$ is

(a) a finite decimal
(b) 1.41421
(c) non-terminating recurring
(d) non-terminating non-recurring

Ans. (d) Since, $\sqrt{2}$ is an irrational number. Also, we know that an irrational number is non-terminating, non-recurring.

Q.9. A rational number equivalent to a rational number $\frac{7}{19}$ is :

(a) $\frac{17}{119}$ (b) $\frac{14}{57}$

(c) $\frac{21}{38}$ (d) $\frac{21}{57}$

Ans. (d) $\frac{7}{19} = \frac{7}{19} \times \frac{3}{3} = \frac{21}{57}$

Q.10. If $\sqrt{10} = 3.162$ then the value of $\frac{1}{\sqrt{10}}$ is :

(a) 0.3162 (b) 3.162
(c) 31.62 (d) 316.2

Ans. (a) 0.3162, $\frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$$= \frac{3.162}{10} = 0.3162$$