

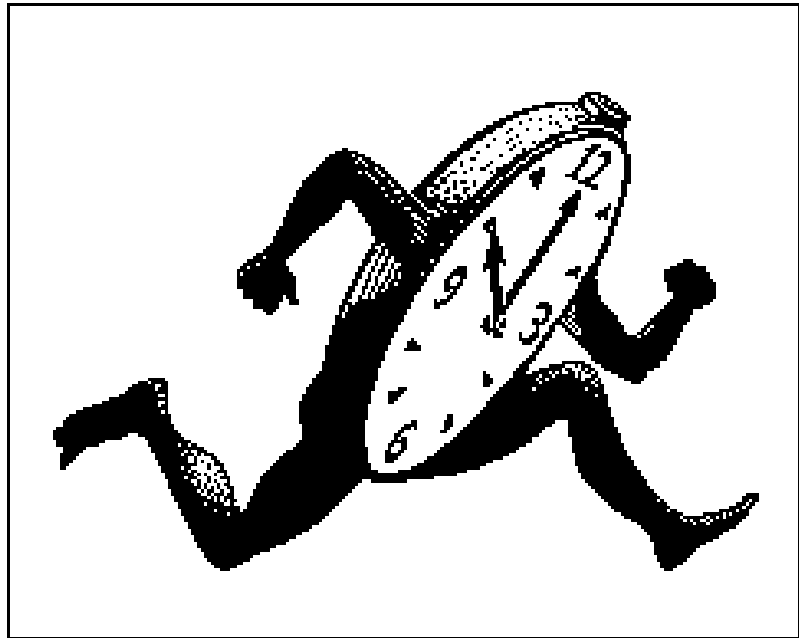
# Announcements

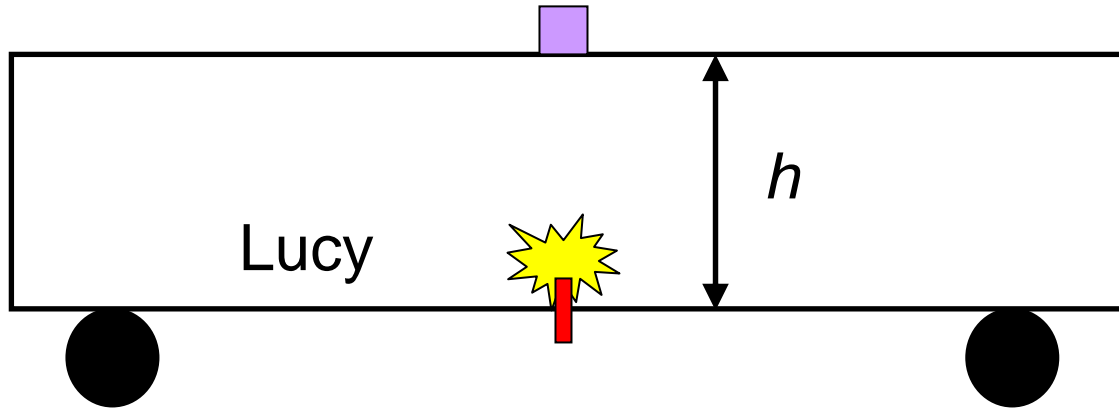
- Homework is due Wed. 10AM (please get it into my mailbox outside the main physics office before I collect it)
- Next week, guest lecturer: Prof. John Price
- Hwk #3 will be on the website before Friday.
- Feedback/online participation will be up later today.

# Time Dilation

Today:

- Clocks run at different rates in different reference frames
- By how much?
- Proper time



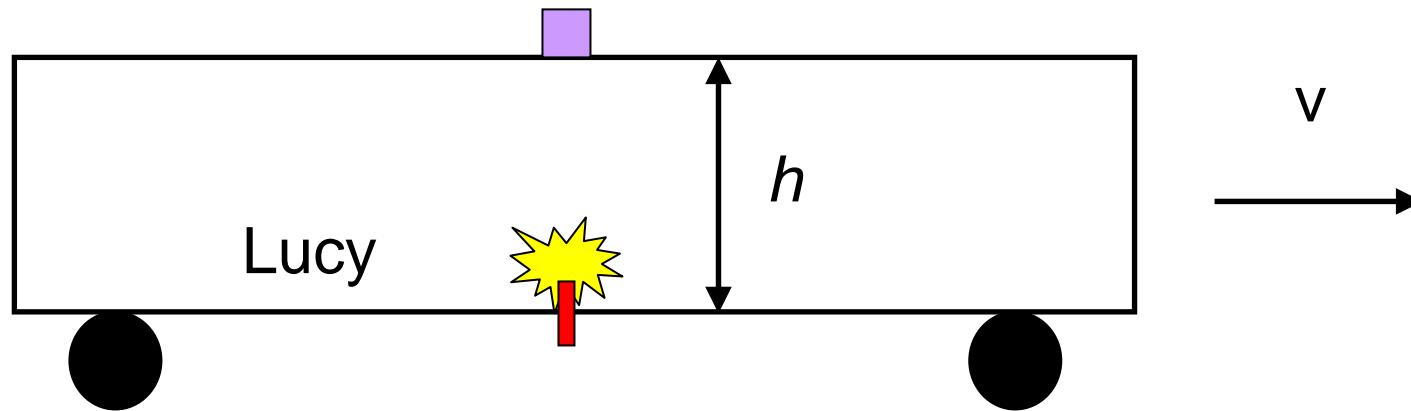


Event 1 – firecracker explodes

Event 2 – light reaches detector

In Lucy's frame, how far apart are these events?

- a)  $h$    b)  $h$    c)  $h$    d) Millard Fillmore



Ethel

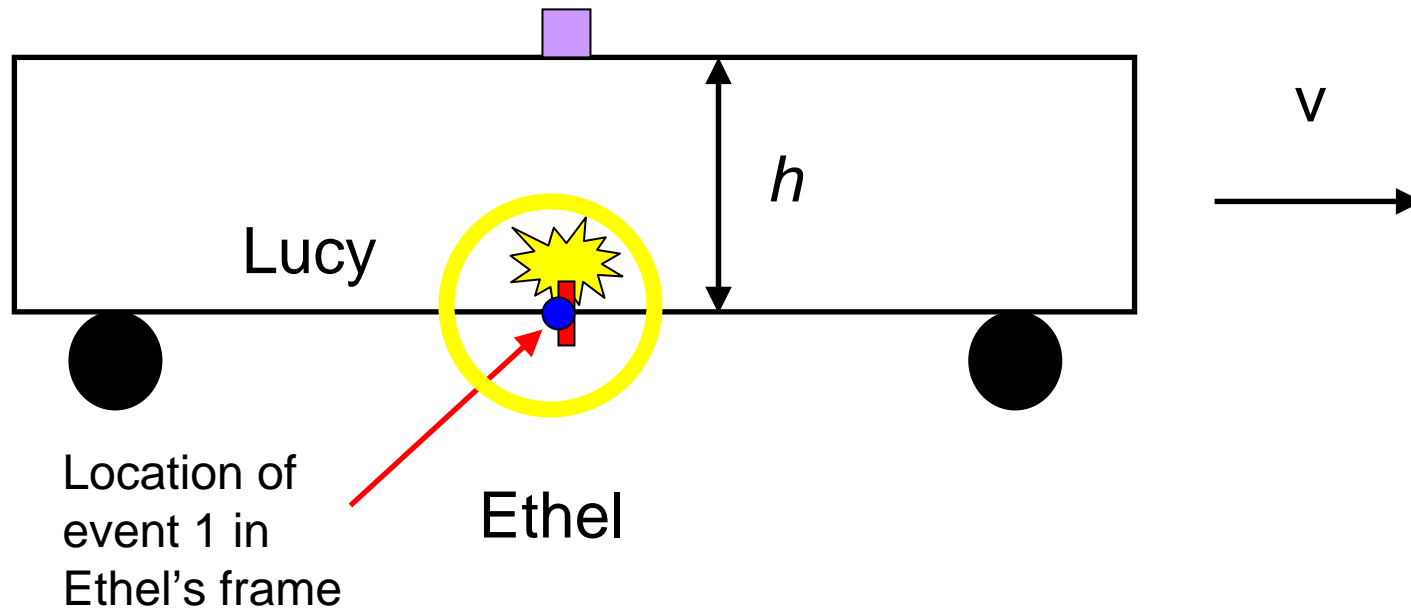
Now Ethel stands by the tracks and watches the train whiz by at speed  $v$ .

Event 1 – firecracker explodes

Event 2 – light reaches detector

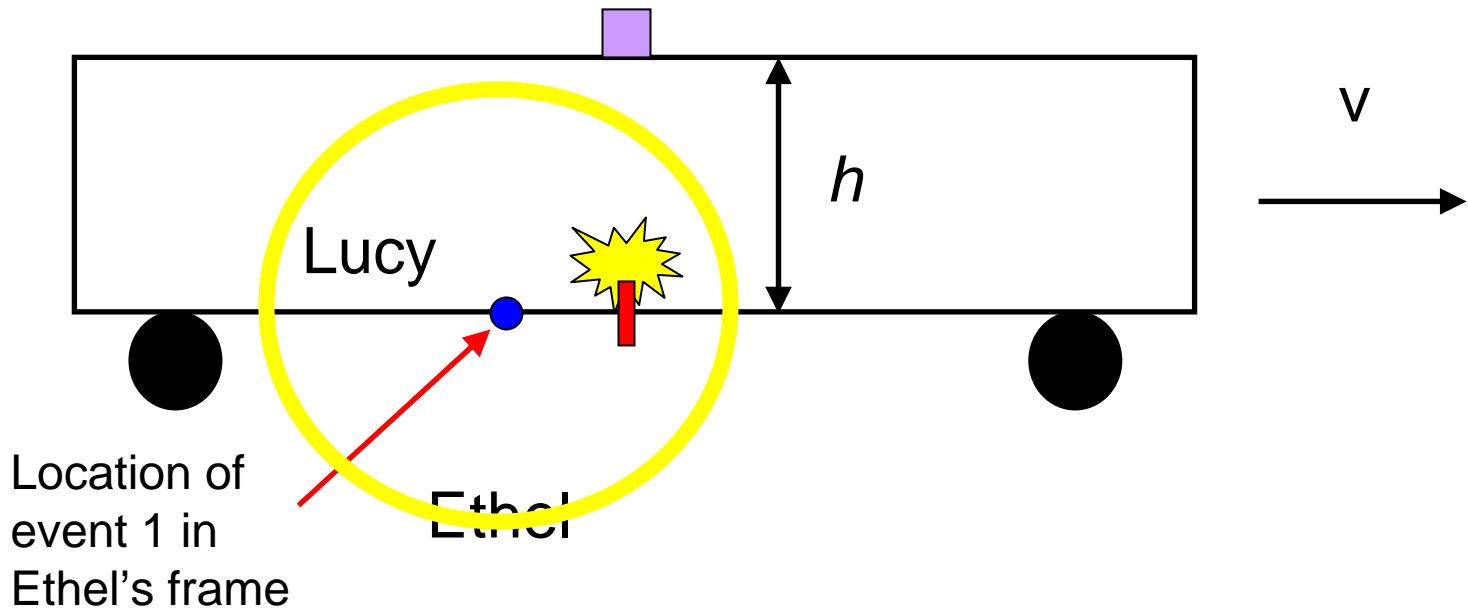
In Ethel's frame, the distance between the two events is

- a) Greater than in Lucy's frame
- b) Less than in Lucy's frame
- c) The same as in Lucy's frame



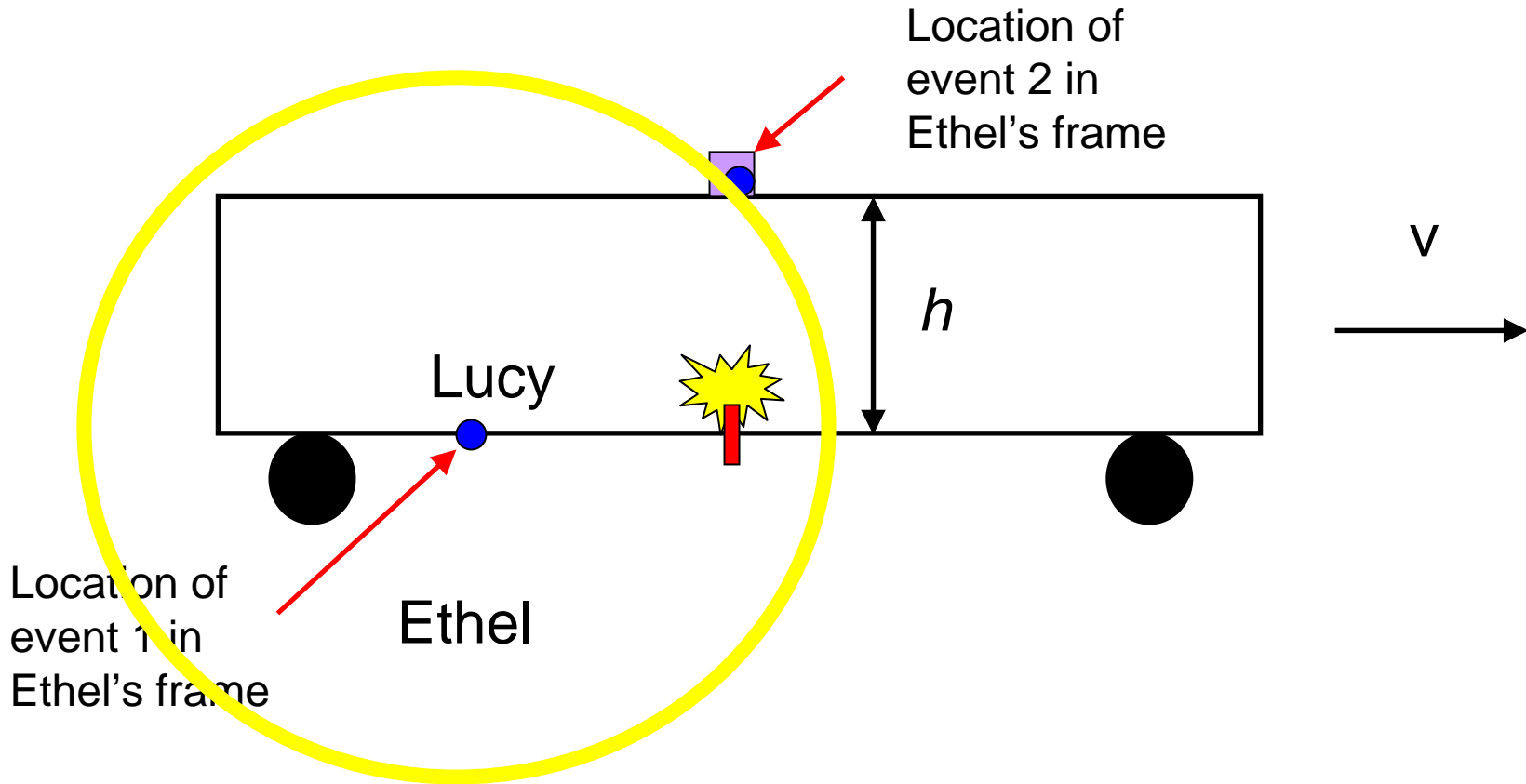
Sure! These events happen at *different  $x$  coordinates* in Ethel's frame.

Event 1 – firecracker explodes



Sure! These events happen at *different  $x$  coordinates* in Ethel's frame.

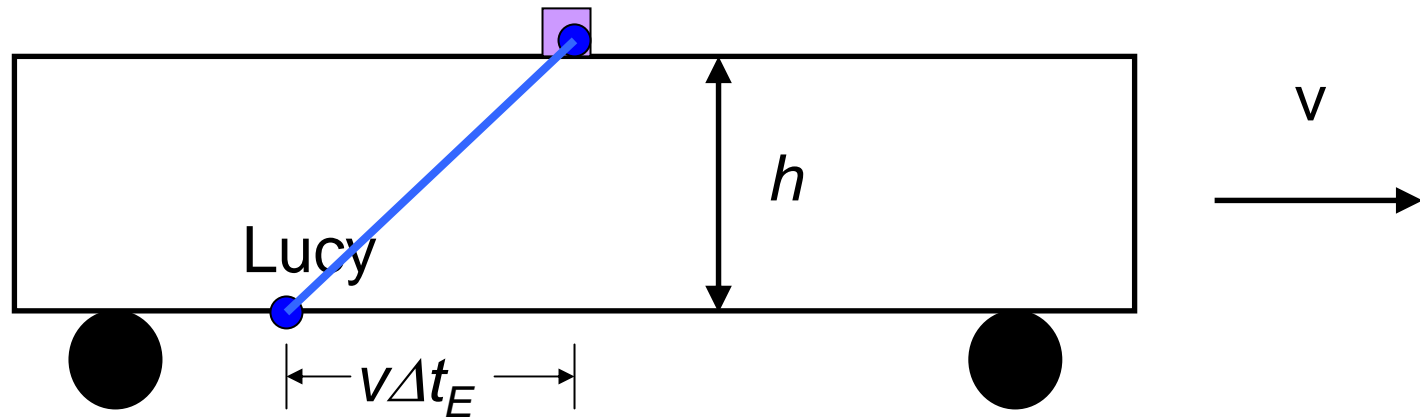
Event 1 – firecracker explodes



Sure! These events happen at *different  $x$  coordinates* in Ethels' frame.

Event 1 – firecracker explodes

Event 2 – light is detected; but the train (and the detector) have moved!



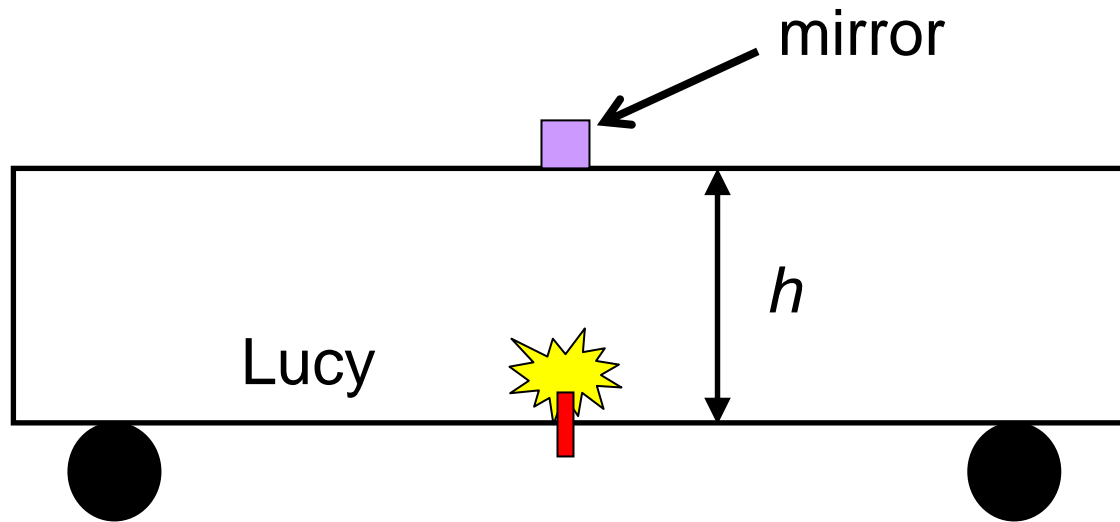
Ethel

If the time between events is  $\Delta t_E$  in Ethel's frame, the train has moved a distance  $v\Delta t_E$ . The distance between the events, in Ethel's frame, is

$$\sqrt{(v\Delta t_E)^2 + h^2}$$

Good old Pythagoras!





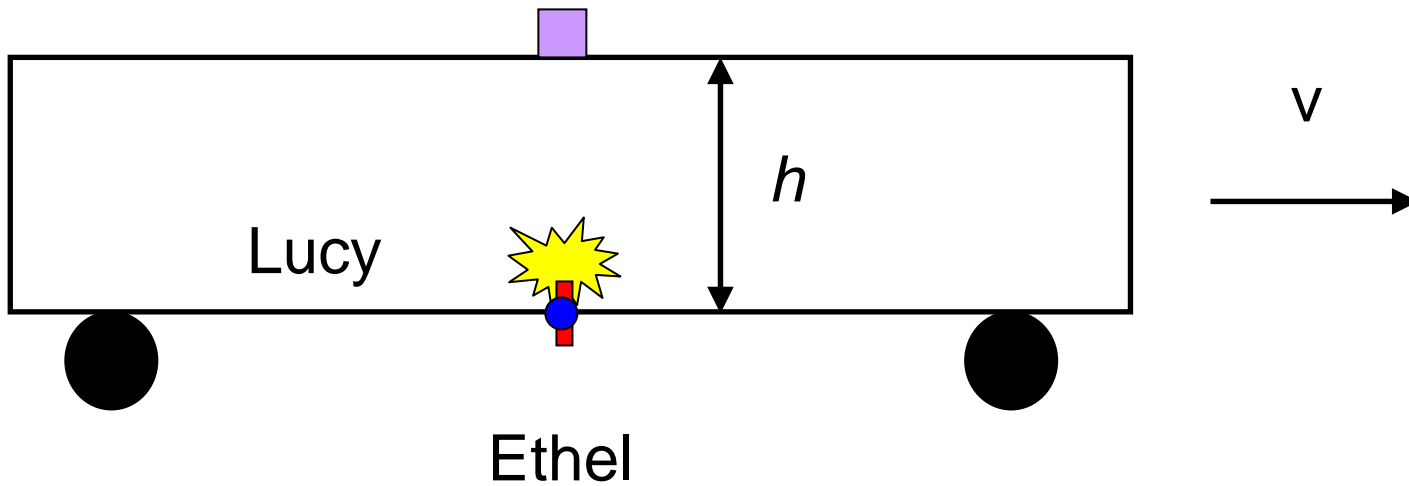
Event 1 – firecracker explodes

Event 2 – light reaches the mirror

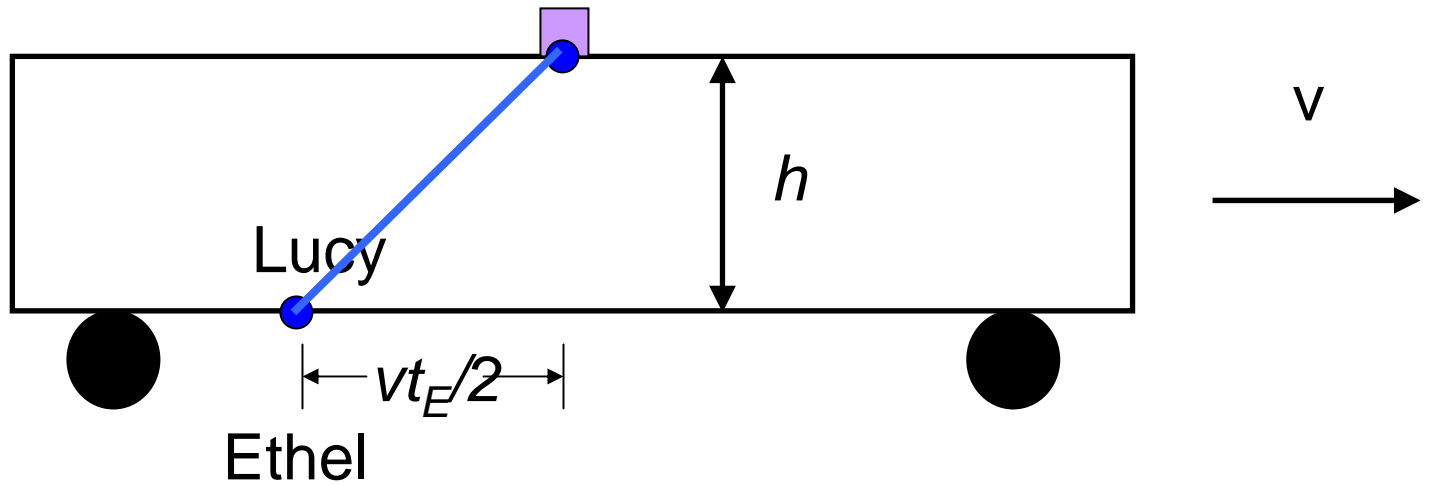
Event 3 – light returns to Lucy

In Lucy's frame, how much time elapses between Event 1 and Event 3?

- a)  $h/c$    b)  $c/h$    c)  $2h/c$    d)  $h/2c$

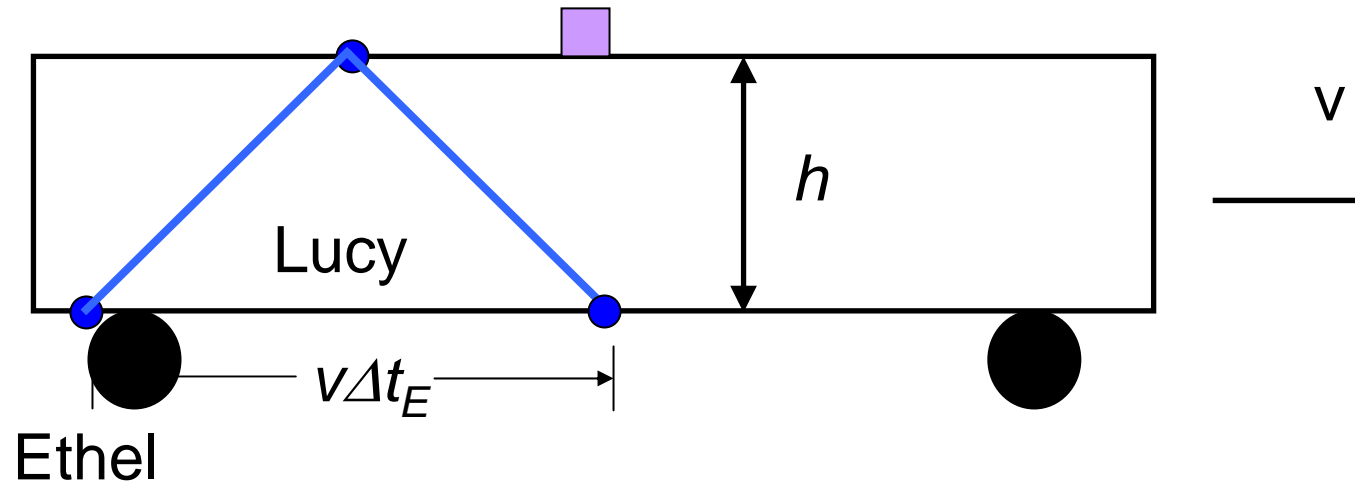


Event 1 – firecracker explodes

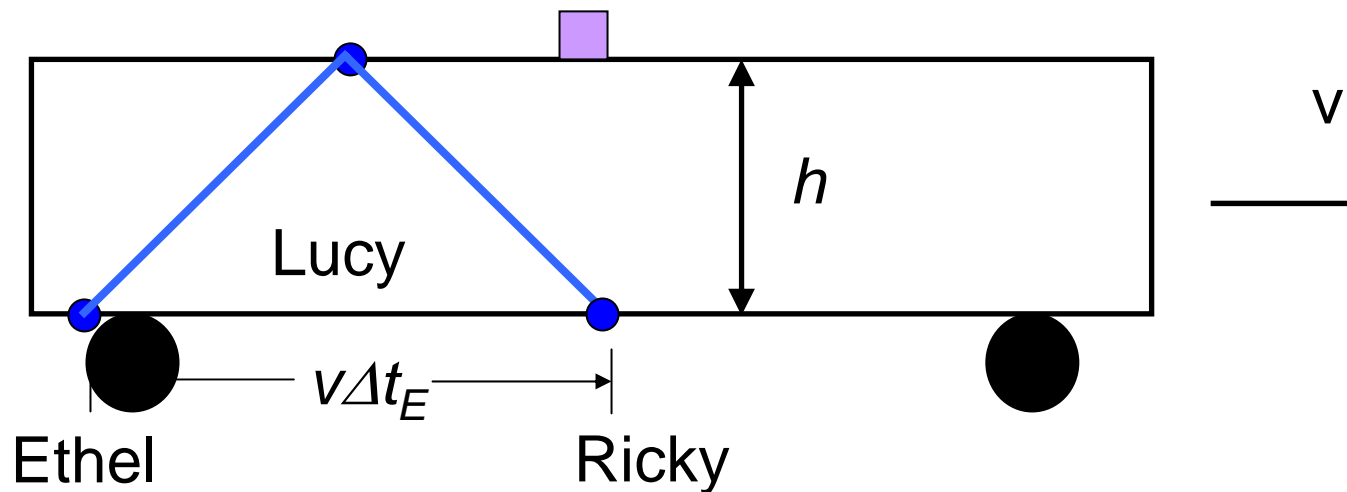


Event 1 – firecracker explodes

Event 2 – light reaches the mirror



- Event 1 – firecracker explodes
- Event 2 – light reaches the mirror
- Event 3 – light returns to Lucy



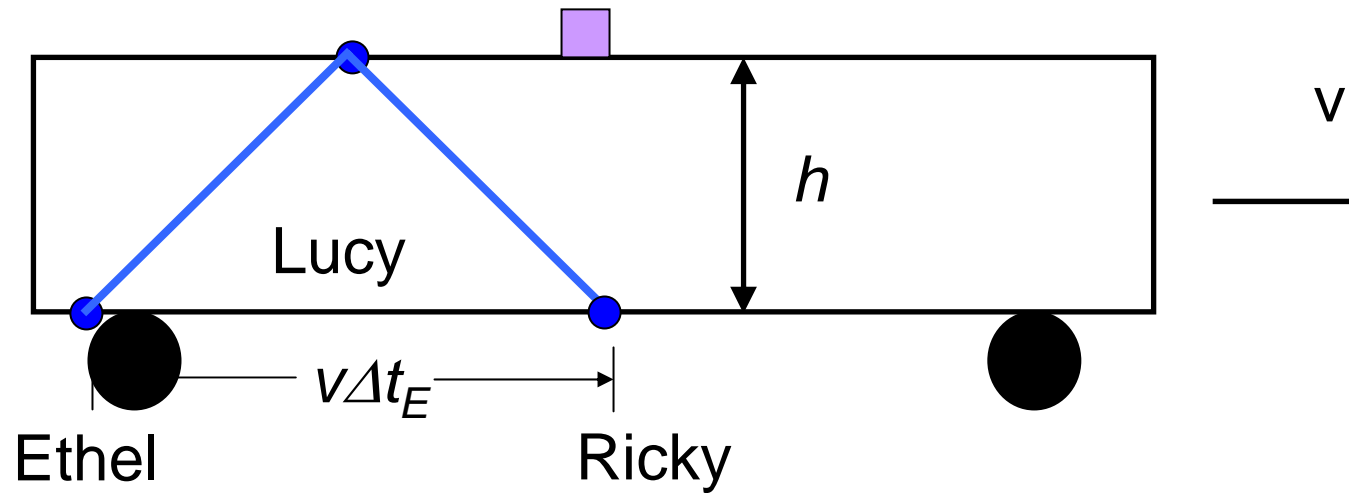
Event 1 – firecracker explodes

Event 2 – light reaches the mirror

Event 3 – light returns to Lucy

In Ethel's frame, how many clocks are required to determine the time between Event 1 and Event 3?

- A) 0   B) 1   C) 2   D) 3   E) none of these



If the time between events is  $\Delta t_E$  in Ethel's frame, the train has moved a distance  $v\Delta t_E$ . The distance between the events, in Ethel's frame, is

$$2\sqrt{\left(v\Delta t_E/2\right)^2 + h^2}$$

Good old Pythagoras!

# Details

In Ethel's frame,

distance between events =(speed of light) X (time between these events)

$$2\sqrt{(v \Delta t_E / 2)^2 + h^2} = c \Delta t_E$$



Algebra

$$\Delta t_E = \frac{2h}{c} \frac{1}{\sqrt{1 - v^2/c^2}} = \Delta t_L \frac{1}{\sqrt{1 - v^2/c^2}}$$

Recall:  $2h = c \Delta t_L$  is the distance between the events in Lucy's frame.

# Final form

Time between events (Ethel) =  $\gamma$  X time between events (Lucy)

$$\Delta t_E = \gamma \Delta t_L \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

According to Ethel, the time between the events is

- a) Greater than
- b) Less than

The time between events according to Lucy.

This is true no matter how fast their relative speed is.



General question: is there something special about these events in Lucy's frame?

a) No

b) Yes

Be prepared to explain your answer.

Answer: Yes! Both events occur at the *same place* in Lucy's frame.

# Proper time

If two events occur at the SAME LOCATION, then the time between them can be MEASURED BY A SINGLE OBSERVER WITH A SINGLE CLOCK (This is the “Lucy time” in our example.) We call the time measured between these types of events the Proper Time,  $\Delta t_0$

Example: any given clock never moves with respect to itself. It keeps proper time in its own frame.

Any observer moving with respect to this clock sees it run slowly (i.e., time intervals are longer).

This is **time dilation**.  $\Delta t = \gamma \Delta t_0$

# A little journey



Vicki



Carol



Carol and Vicki are identical twins. While Vicki stays on Earth, Carol departs for the star Sirius, 8 light-years away, traveling at a speed  $v = 0.8 c$  (Note  $\gamma = 5/3$ ). According to observers in **Vicki's** frame, how long does the trip take?

- a) 6 years      b) 8 years      c) 10 years      d) 16.67 years

# A little journey



Vicki



Carol



Carol and Vicki are identical twins. While Vicki stays on Earth, Carol departs for the star Sirius, 8 light-years away, traveling at a speed  $v = 0.8 c$  (Note  $\gamma = 5/3$ ). According to **Carol**, how long does the trip take?

- a) 6 years    b) 8 years    c) 10 years    d) 16.67 years

Are you sirius?

# A little journey



Vicki



Carol



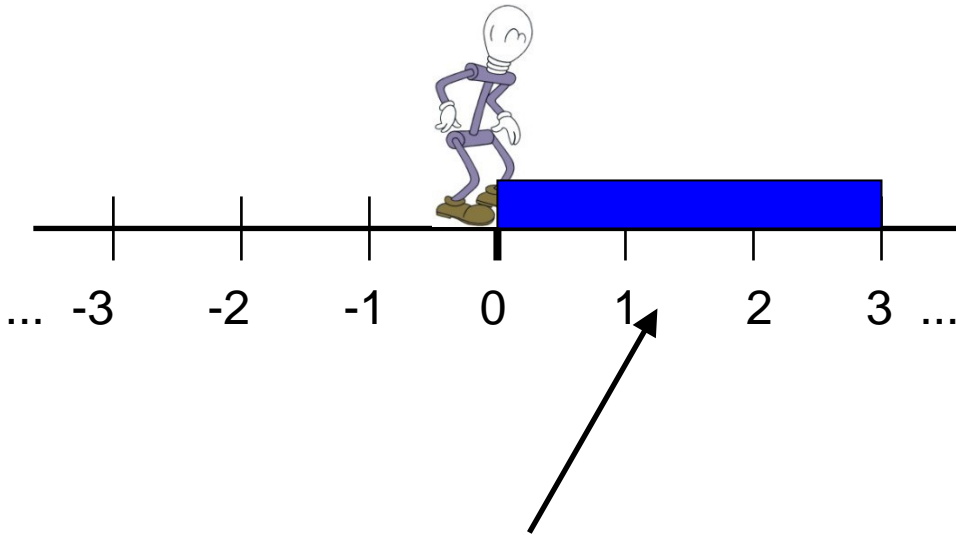
Why? Because Carol's clock is present at both the events:  
Carol is at Earth.  
Carol is at Sirius.

So if  $\Delta\tau$  is Carol's proper time between these events, and  $\Delta t$  is the time in the Earth-Sirius system, we have

$$\Delta\tau = \frac{\Delta t}{\gamma} = \frac{10y}{5/3} = 6y$$

Follow the proper time!

# Length of an object



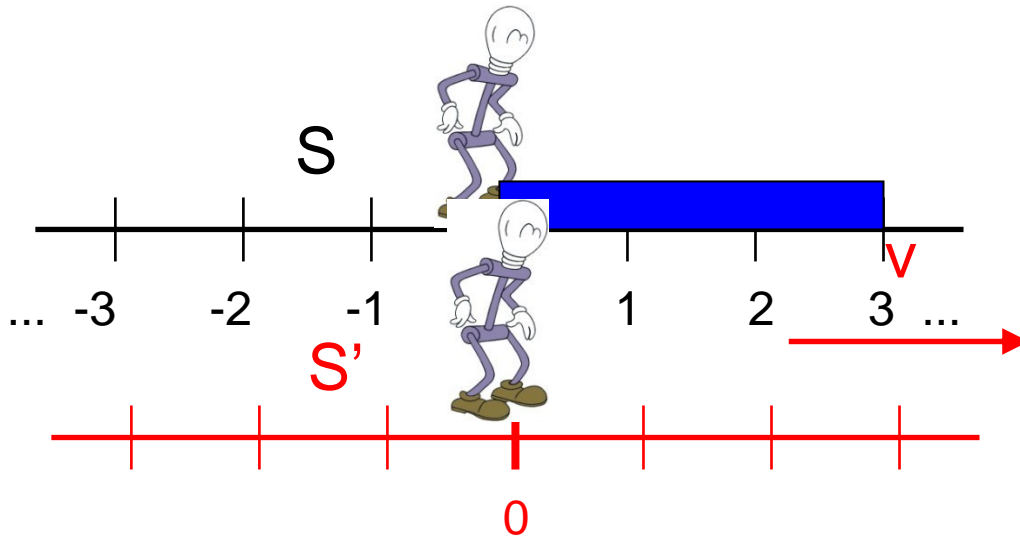
This length, measured in the stick's rest frame, is its **proper length**.

This stick is 3m long. I measure both ends at *the same time* in my frame of reference.

Or not. It doesn't matter, because the stick isn't going anywhere.

But as we know, “at the same time” is relative – it depends on how you're moving.

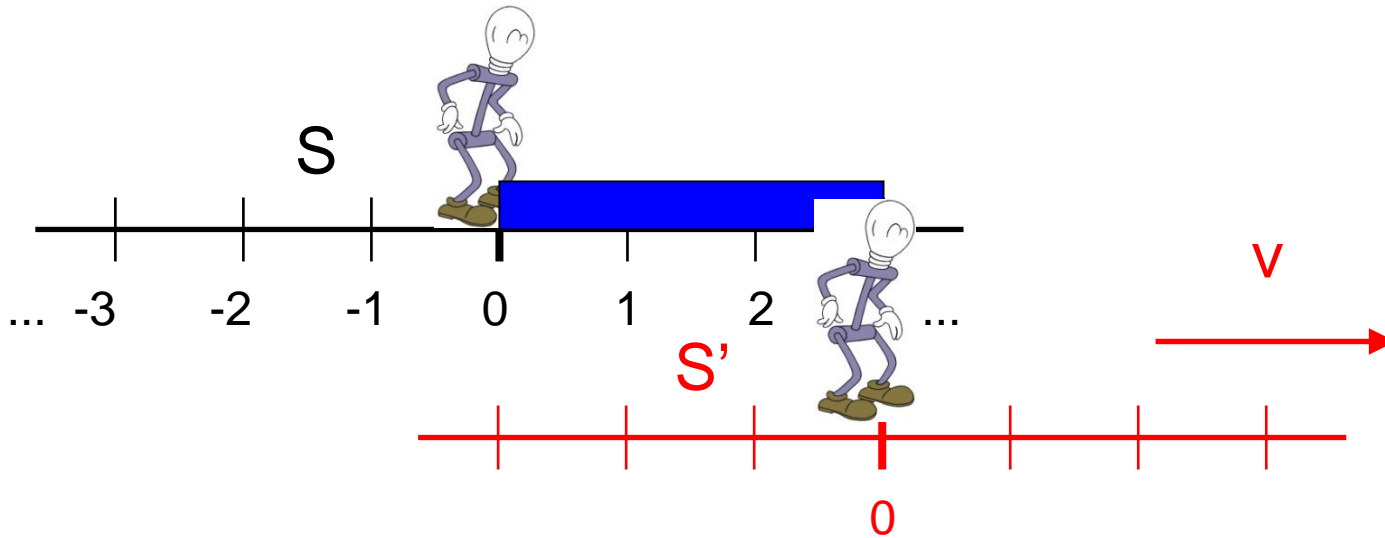
# Length of an object



Another observer comes whizzing by at speed  $v$ . This observer measures the length of the stick, *and keeps track of time.*

Event 1 – Origin of  $S'$  passes left end of stick.

# Length of an object

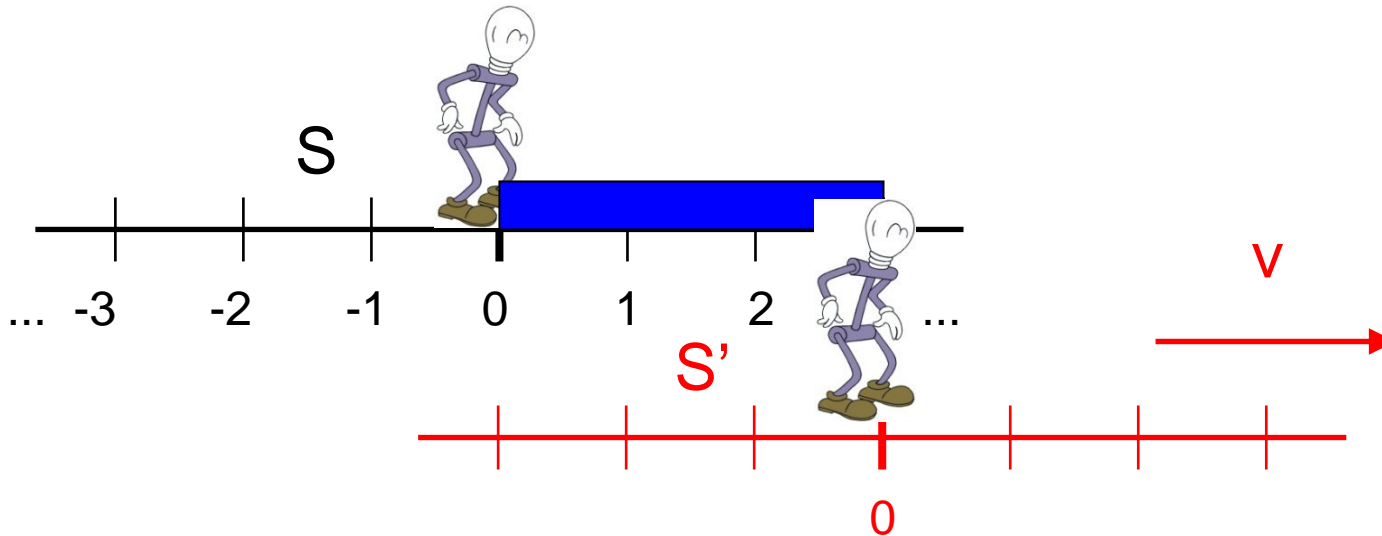


Event 1 – Origin of  $S'$  passes left end of stick.

Event 2 – Origin of  $S'$  passes right end of stick.



# Length of an object



Event 1 – Origin of  $S'$  passes left end of stick.

Event 2 – Origin of  $S'$  passes right end of stick.

How many observers are needed in  $S$  to measure the time between events? A) 0 B) 1 C) 2 D) 57

# A little math

In frame S:

length of stick =  $L$  (this is the proper length)

time between measurements =  $\Delta t$

speed of frame  $S'$  is  $v = L/\Delta t$

In frame  $S'$ :

length of stick =  $L'$  (this is what we're looking for)

time between measurements =  $\Delta t'$

speed of frame  $S$  is  $v = L'/\Delta t'$

Q: a)  $\Delta t = \gamma \Delta t'$  or b)  $\Delta t' = \gamma \Delta t$

Follow the proper time!

# A little math

Speeds are the same (both refer to the relative speed).

And so

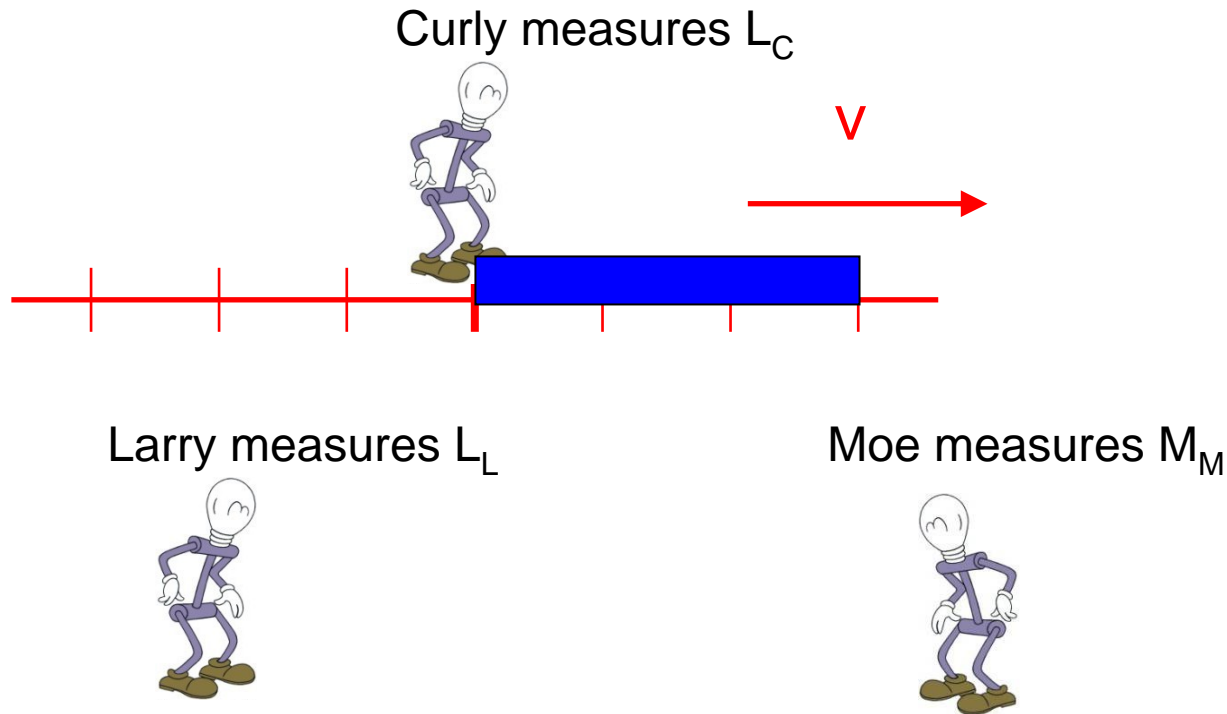
$$v = \frac{L}{\Delta t} = \frac{L'}{\Delta t'} = \frac{\gamma L}{\Delta t}$$

$$L' = \frac{L}{\gamma}$$

Length in moving frame

Length in stick's rest frame  
(proper length)

Length contraction is a consequence of time dilation  
(and vice-versa).



Curly runs by real fast with a stick he knows to be of length  $L_C$ . Larry and Moe are both standing on the ground and each measures the stick as it goes by. How are the three measurements related?

a)  $L_C < L_L < L_M$

b)  $L_C > L_L > L_M$

c)  $L_C = L_L = L_M$

d)  $L_C < L_L = L_M$

e)  $L_C > L_L = L_M$