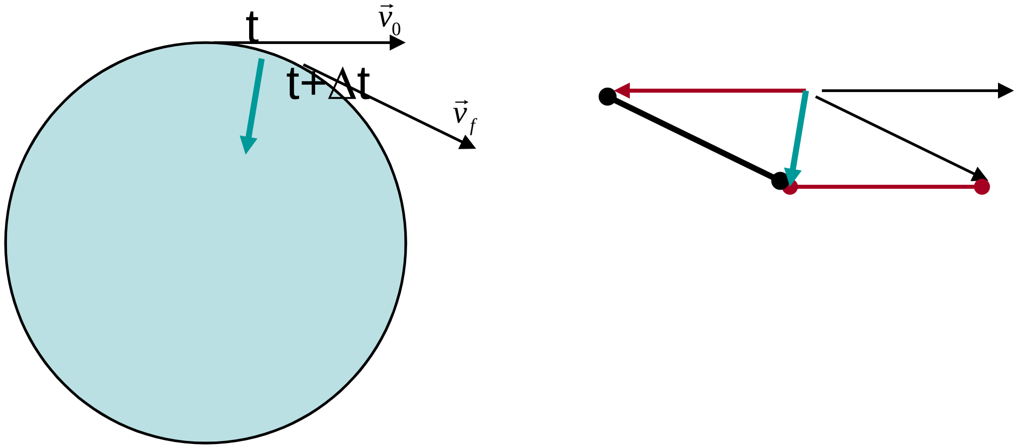
Theory of Relativity

Centripetal Acceleration

$$\vec{a} = \Delta\vec{v} / \Delta t = (\vec{v}_f - \vec{v}_0) / \Delta t$$

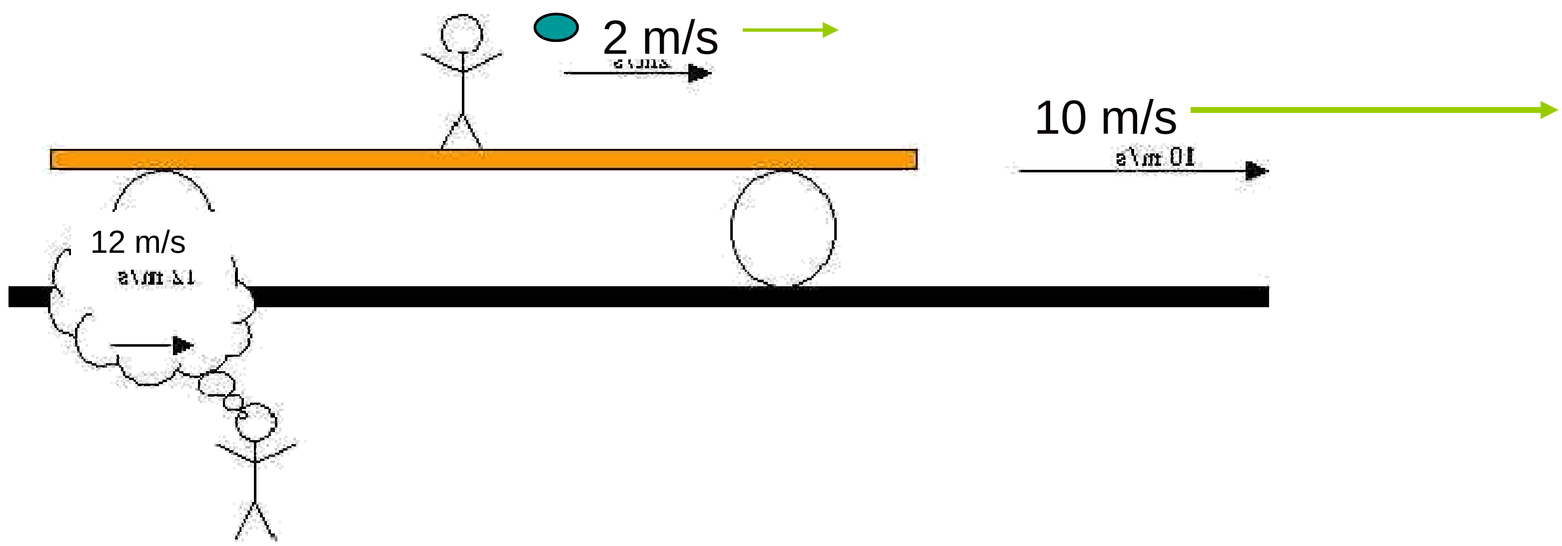


Concept Summary

- Galilean Relativity and Transformation
- Maxwell’s Equations – speed of light
- Ether – MM experiment
- Einstein’s postulates
- Simultaneity
- Time dilation
- Length contraction
- Lorentz transformation
- Examples
- Relativistic Doppler effect

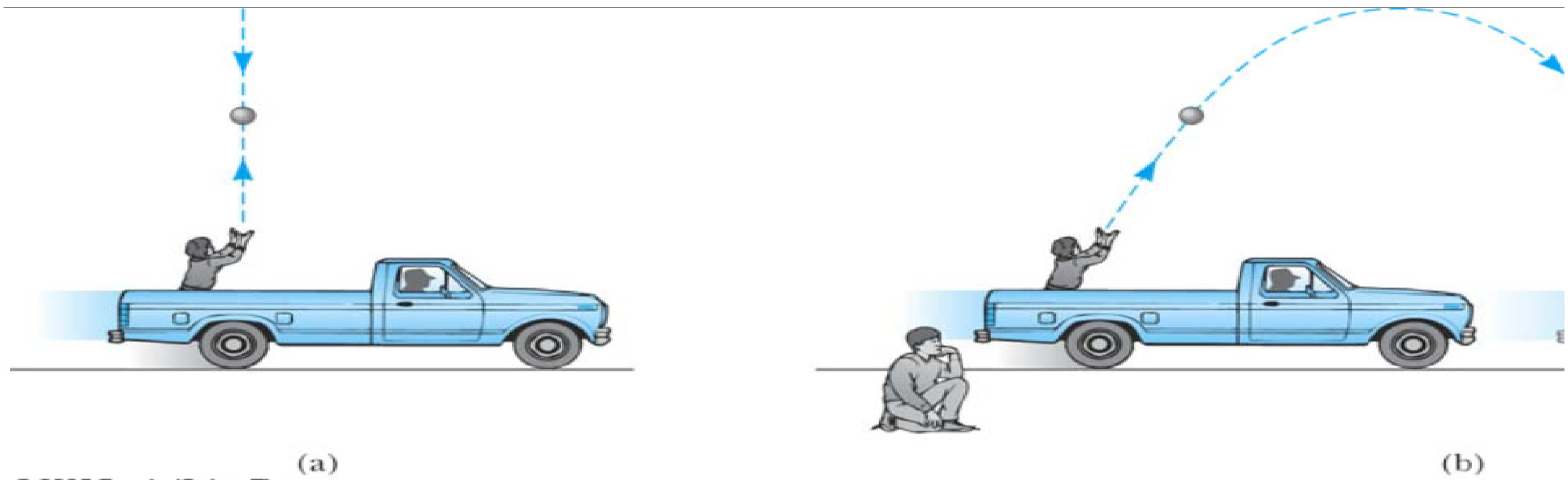
Galilean Relativity

- "Relativity" refers in general to the way physical measurements made in a given inertial frame are related to measurements in another frame.
- An inertial observer is one whose rest frame is inertial
- A quantity is invariant if all inertial observers obtain the same value
- Under Galilean relativity, measurements are transformed simply by adding or subtracting the velocity difference between frames:
- v_{ball}(measured on ground)=v_{train} (measured on ground)+v_{ball}(measured on train)
  12 m/s = 10m/s + 2 m/s
- V_{ball}(measured on train)=v_{ground}(measured on train)+ v_{ball}(measured on ground)
  2 m/s = –10m/s + 12 m/s



Experiment at rest

Experiment in moving frame



Same result. Ball rises and ends up in the thrower's hand. Ball in the air the same length of time.

Experiment looks different from ground observer (parabolic trajectory, speed as a function of time) and observer on the truck.

However, they both agree on the validity of Newton's laws.

Fig. 1-1, p. 4

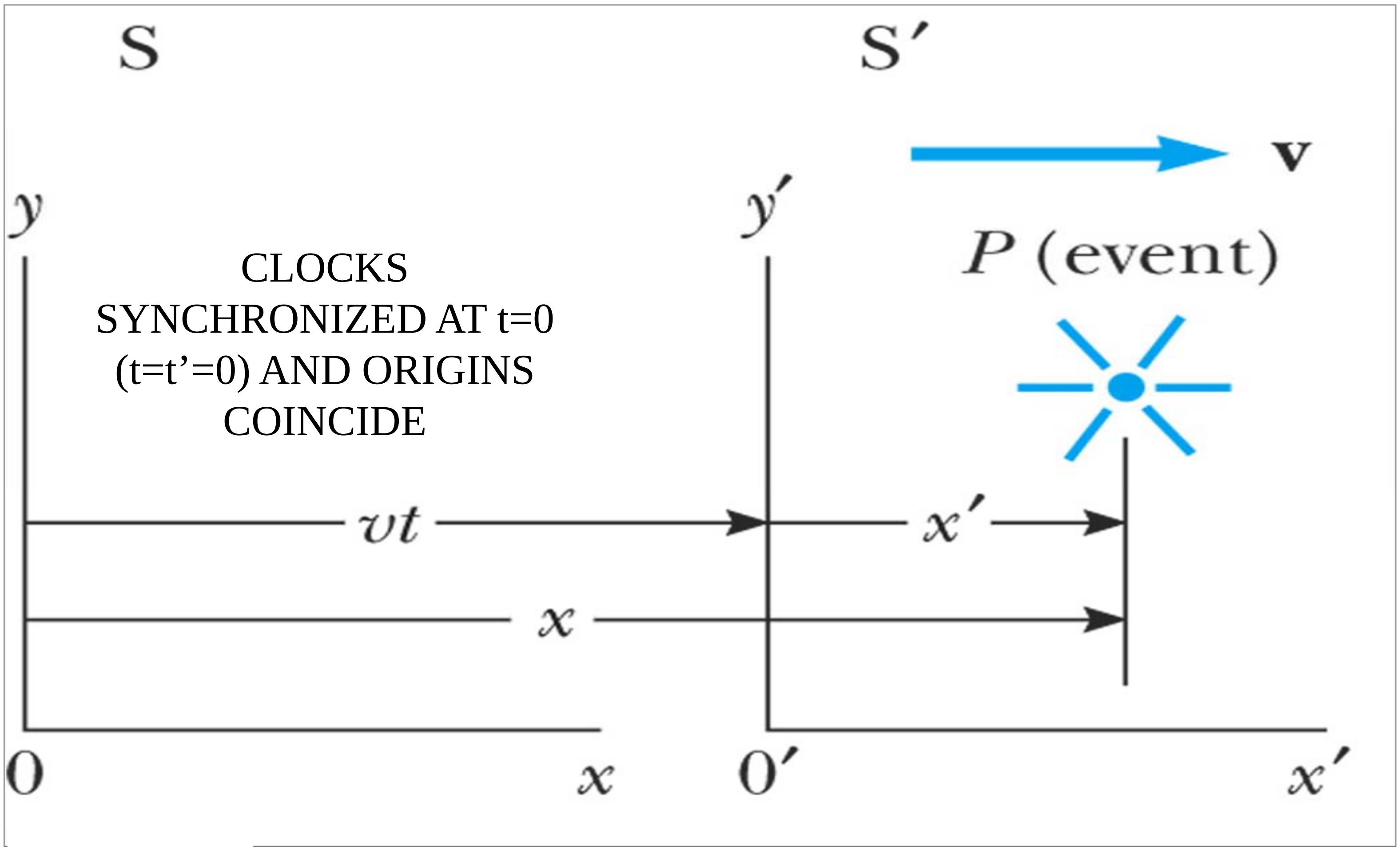
Event – Galilean transformation

$$x = x' + vt'$$

$$y = y'$$

$$t = t'$$

Laws of mechanics invariant under Galilean transformation



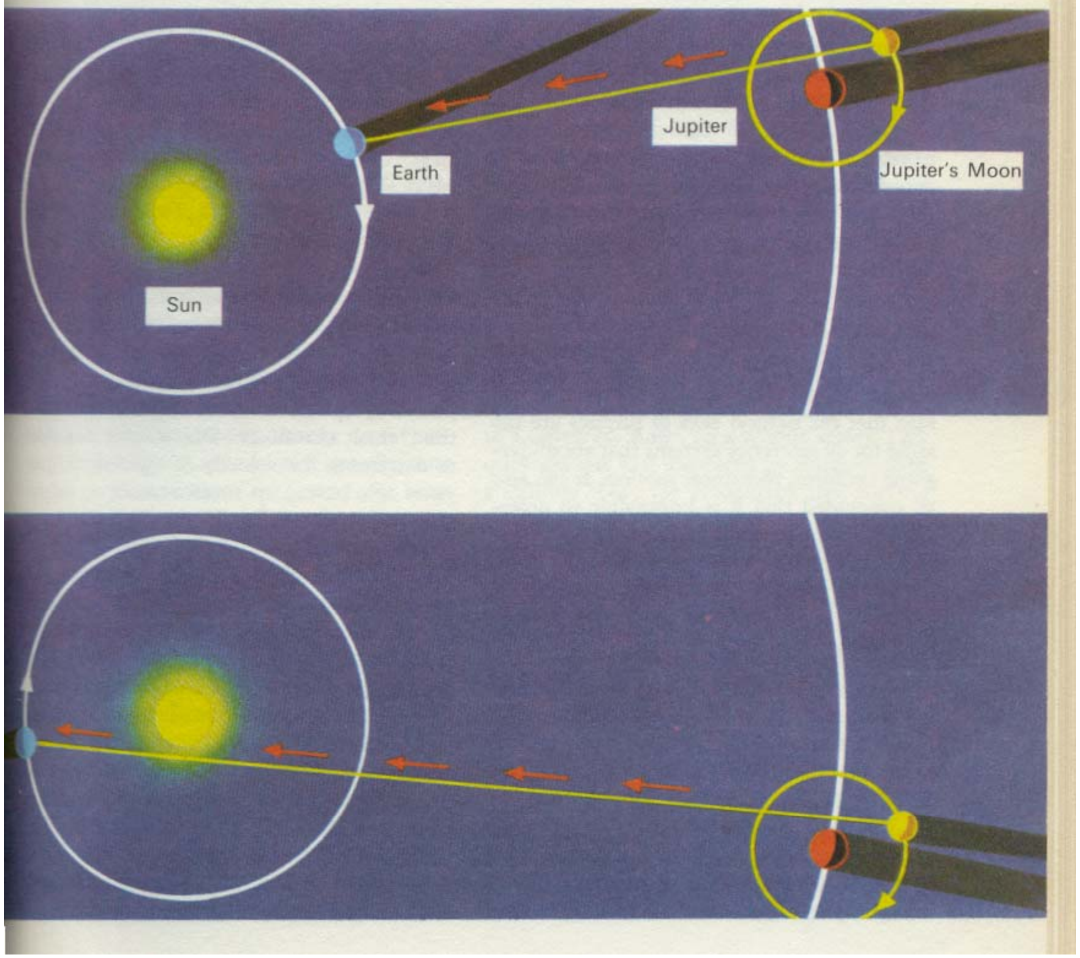
The Speed of Light

But at what frame is its value

$3x10^5$ km/sec ?

Ether

The eclipse of one of Jupiter's moons occurs earlier than calculated when Jupiter is close to the Earth. However, it occurs later if the Earth and Jupiter are far away from each other. About 300 years ago, the Dane Olaf Römer calculated the speed of light on the basis of this irregularity.



Electromagnetic waves

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations).

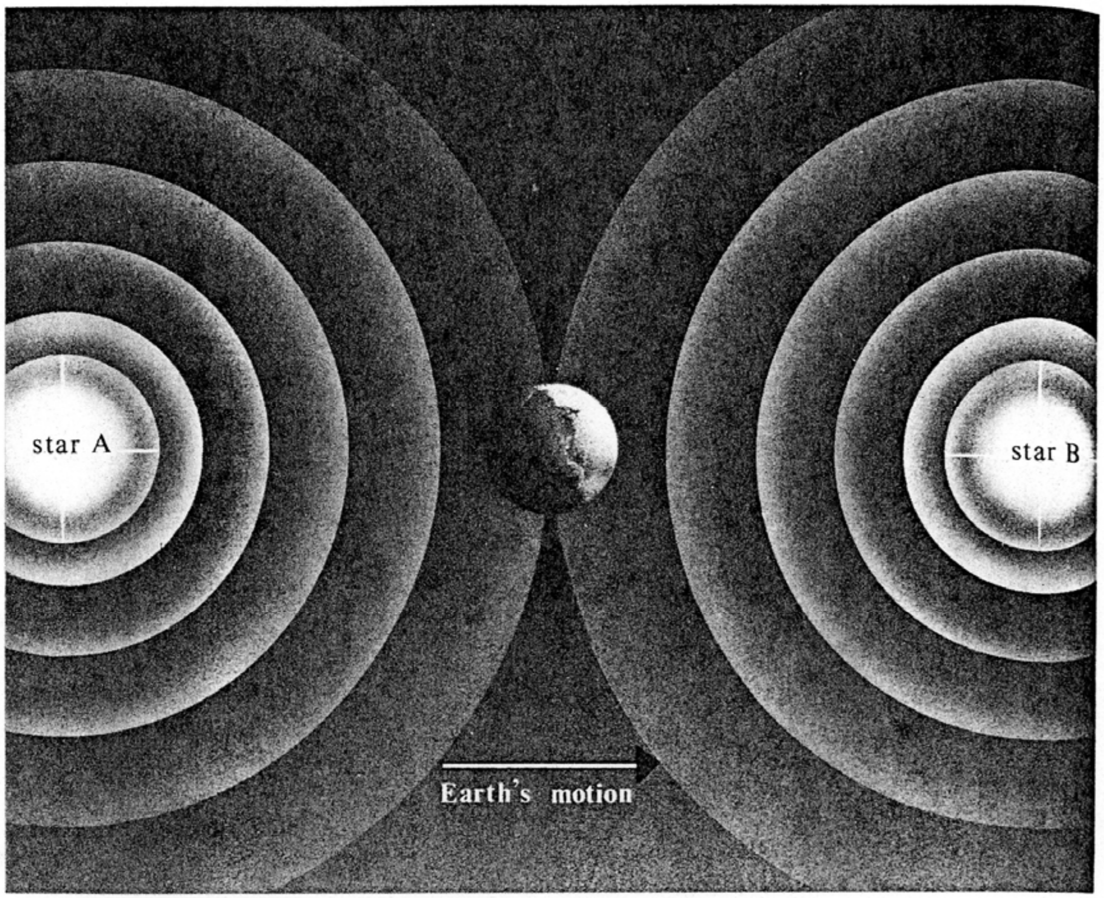
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / c + (1/c) \partial \mathbf{E} / \partial t$$

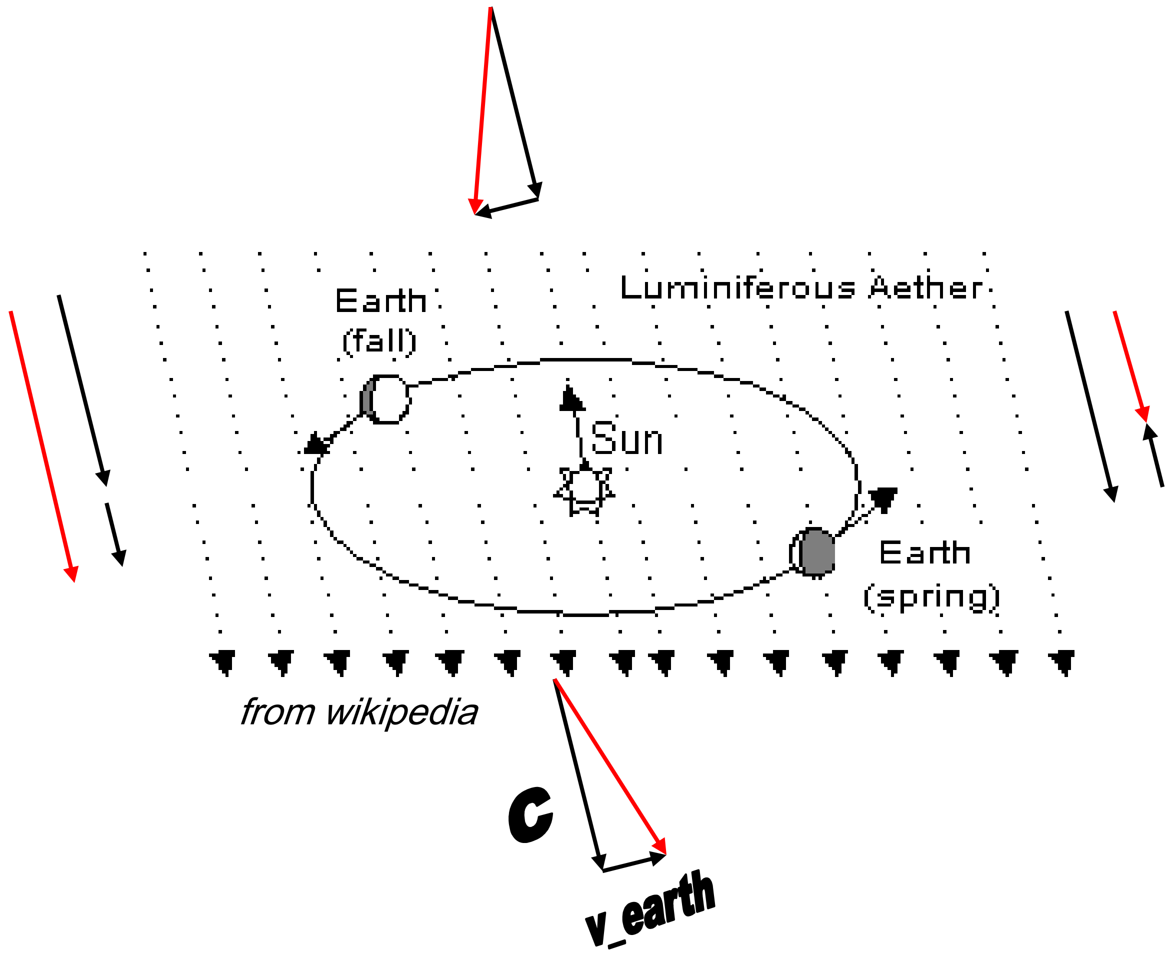
ETHER



Aether drift theory held that if the velocity of light was constant relative to a stationary, all-pervading aether, then when the earth in its orbit was moving away from star *A* and toward star *B*, the observed speed of the light coming from star *B* would be higher than that of the light coming from star *A*.

From T. Ferris : "Coming of Age in the Milky Way"

Light must travel through a medium: hypothesize that a “luminiferous ether” exists



Earth is moving with respect to the ether (or the ether is moving with respect to the earth), so there should be some directional/season dependent change in the speed of light as observed from the reference frame of the earth.

MM experiment gave negative result

NO ETHER – NO PREFERRED FRAME

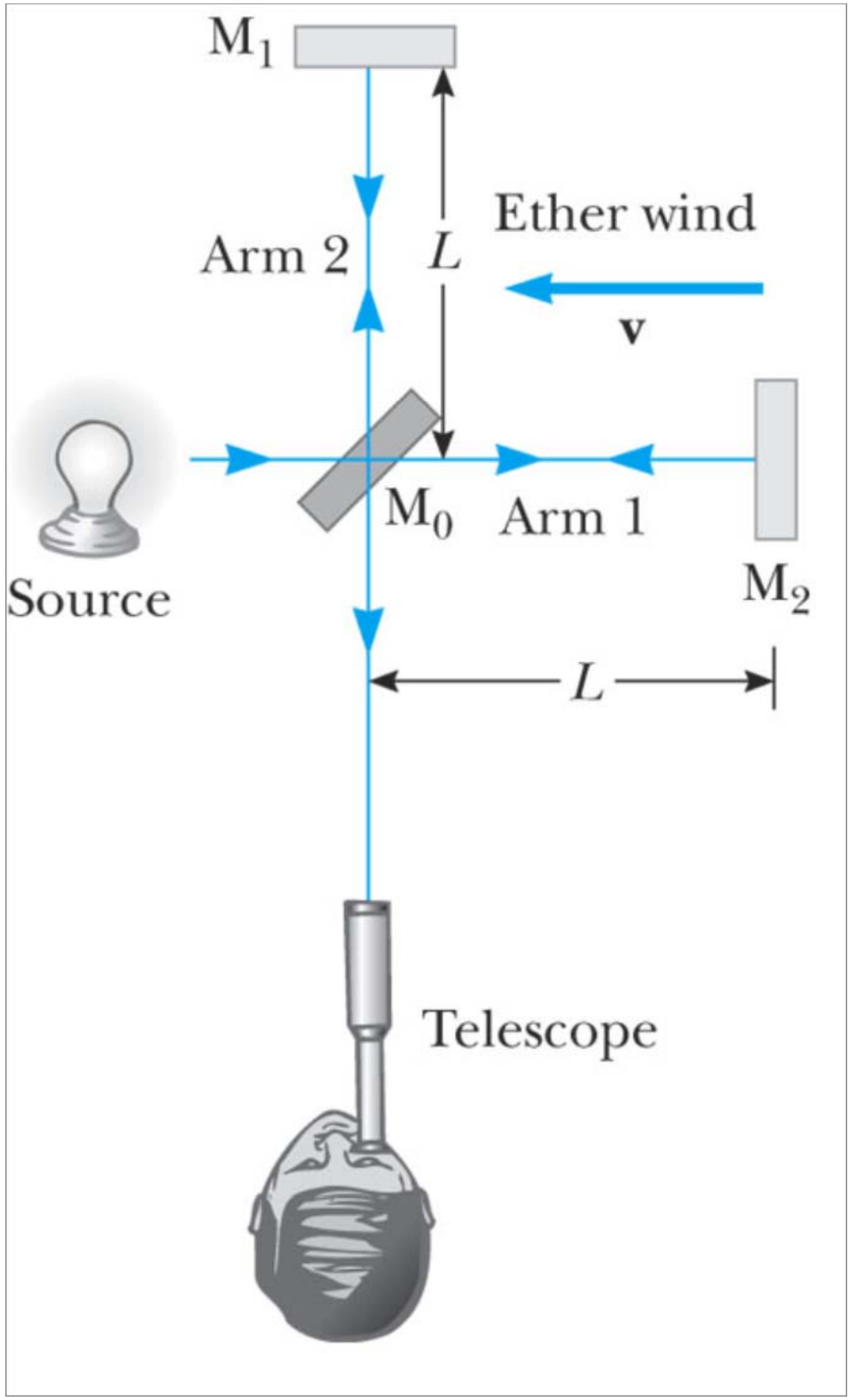
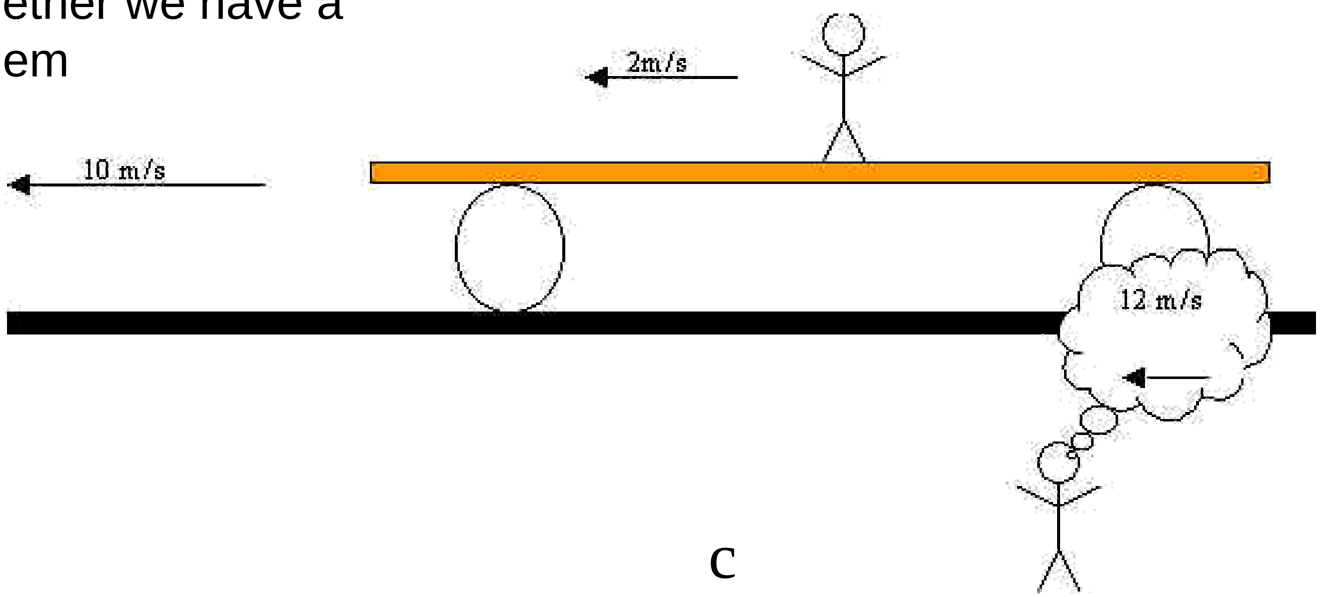
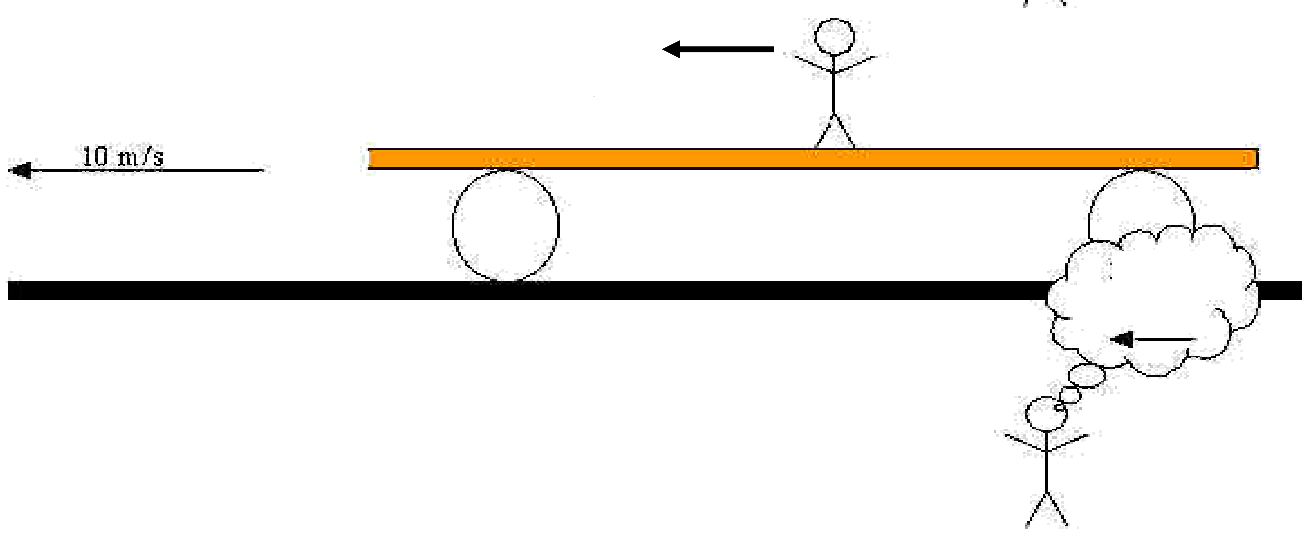


Fig. 1-4, p. 8

THE SPEED OF LIGHT PROBLEM

If no ether we have a problem





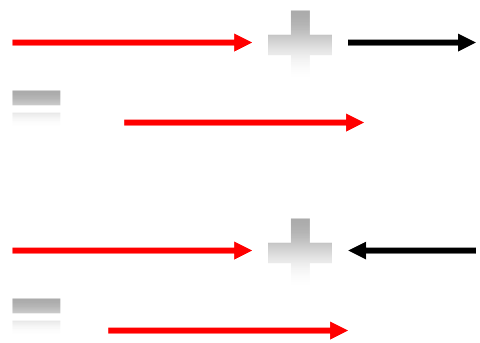
Einstein enters the picture…

- Albert Einstein
  - Didn't like idea of Ether
  - Threw away the idea of Galilean Relativity
  - Came up with the two "Postulates of Relativity"

- **Postulate 1** – The laws of nature are the same in all inertial frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all inertial frames of reference.

The Solution???

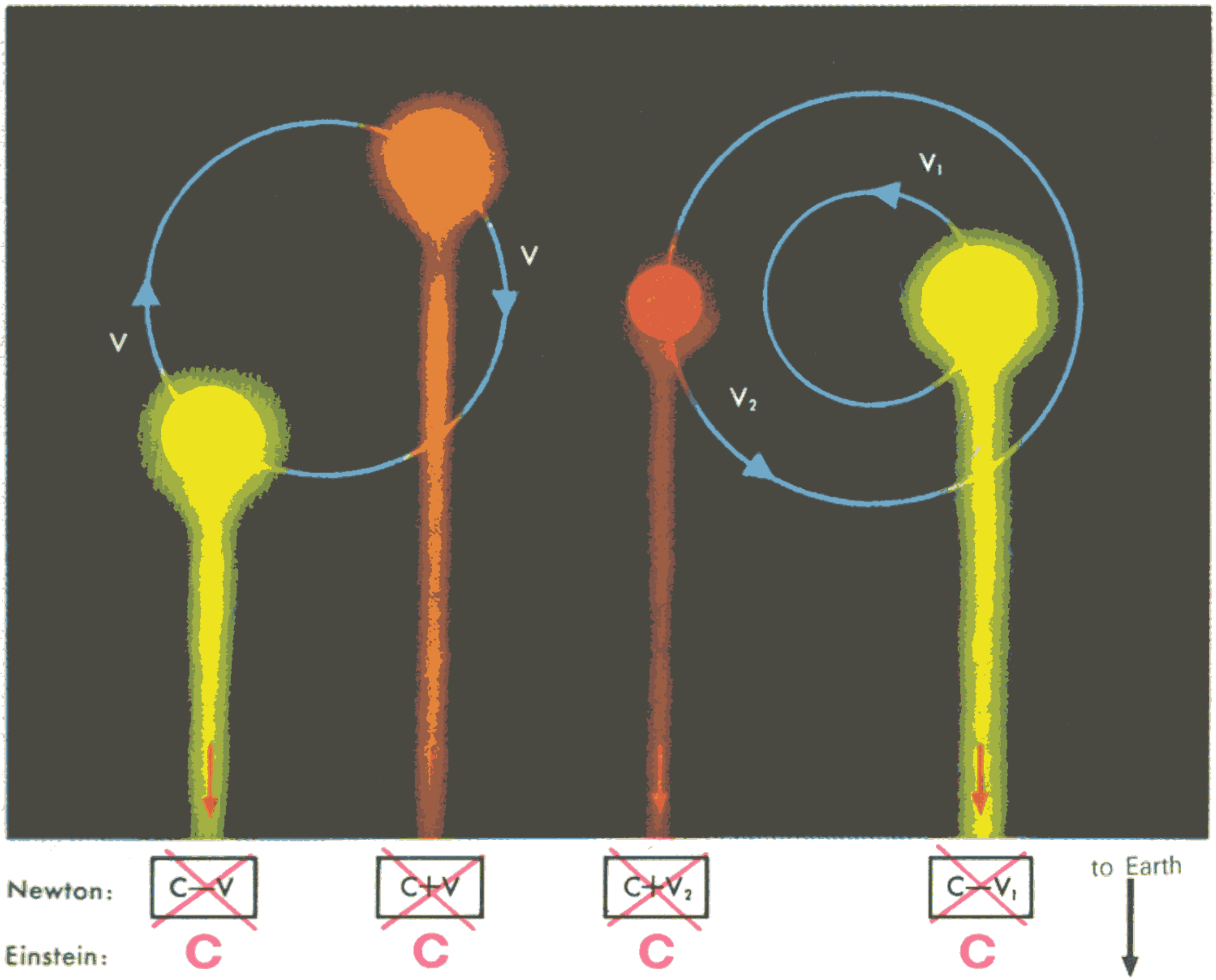
The speed of light in vacuum has the same value, c=300000000 m/s, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.



All the laws of physics have the same form in all inertial reference frames.

Alright…we know that Newtonian mechanics worked in all inertial reference frames under Galilean transformations, but does the same hold true for Maxwell's equations of electromagnetism?

INVARIANCE OF SPEED OF LIGHT



Observations of double stars confirm Einstein's new formula for addition of velocities, in which the velocity of light represents a limiting value.

TRUTH AND CONSEQUENCES

WHAT IS SPEED ?

WHAT DISTANCE AN OBJECT WILL TRAVEL IN A GIVEN DURATION OF TIME V=DX/DT

DISTANCE IS A NOTION ABOUT SPACE – HOW MUCH SPACE IS BETWEEN TWO POINTS

DURATION IS A NOTION ABOUT TIME – HOW MUCH TIME ELAPSES BETWEEN EVENTS

SPEED IS A SPACE-TIME NOTION – CONSTANCY OF SPEED OF LIGHT REQUIRES THAT WE MODIFY CONVENTIONAL CONCEPTS OF SPACE AND TIME

The radical consequences

$$\textbf{Speed} = \frac{\textbf{distance traveled}}{\textbf{time elapsed}}$$

If the speed of light is a constant…then…length and time must be variables??

These effects are known as length contraction and time dilation.

How come you never noticed this before, and how come *most of the time* I can get away with Galilean transformations in your calculations?

speed of light = 670 616 629 miles per hour

Most of the time the speed of the object whose motion you are calculating is so slow relative to the speed of light that the discrepancy due to relativity is negligible. (Most, but not all of the time)

REFERENCE FRAME -> GRID (3D) + SET OF CLOCKS

CLOCKS SYNCHRONIZED

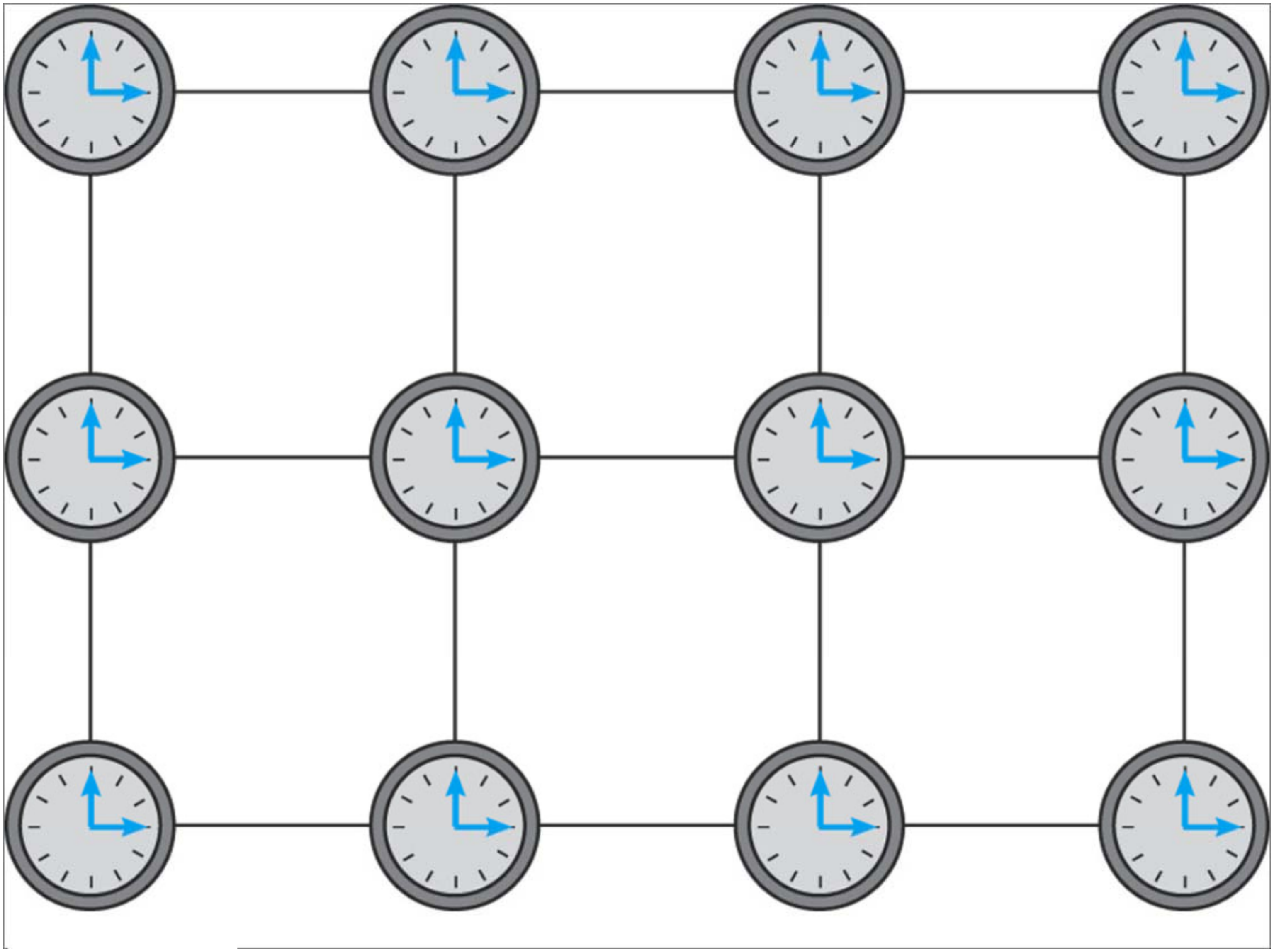
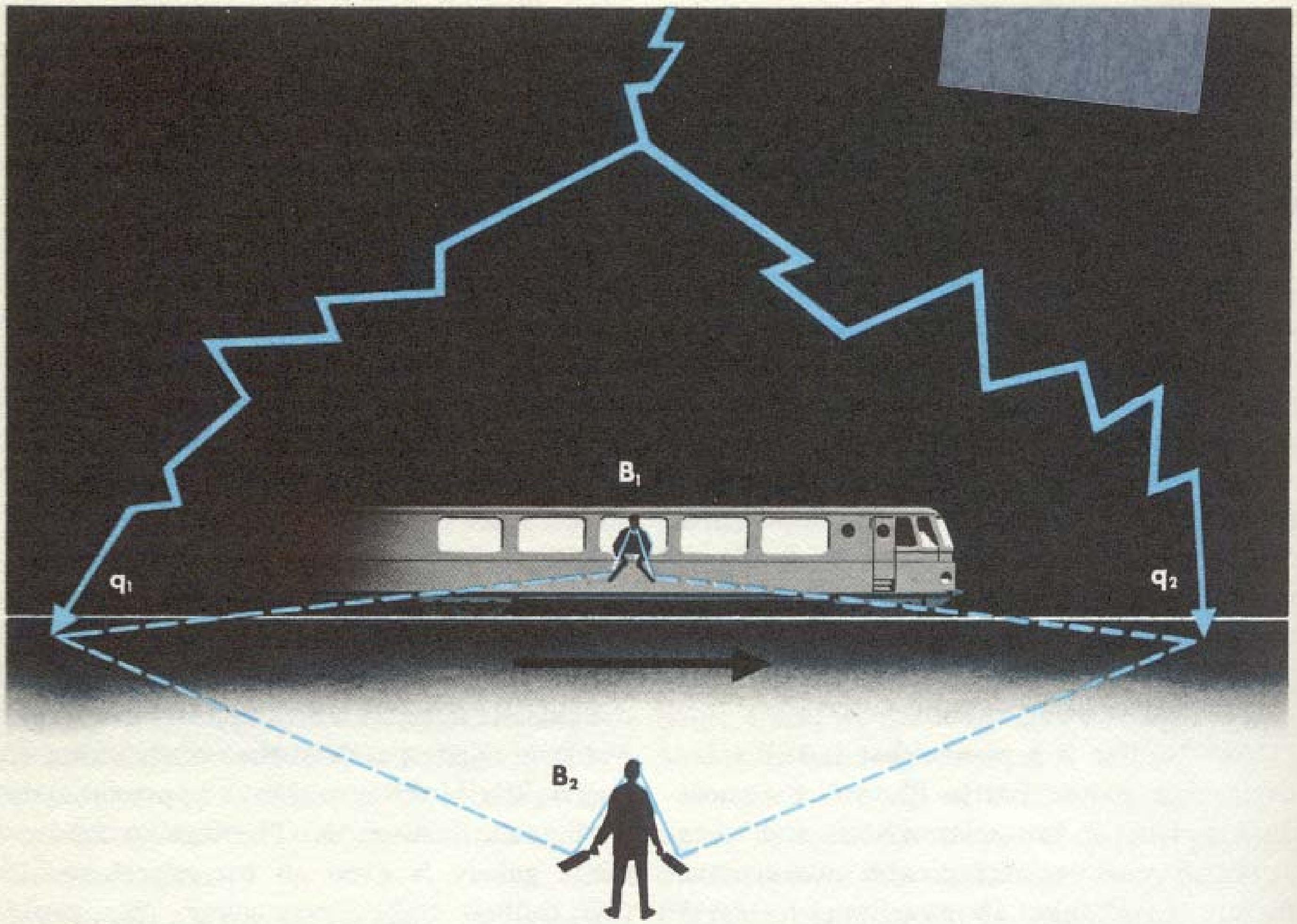


Fig. 1-8, p. 13

SIMULTANEITY

- NEWTON -> UNIVERSAL TIMESCALE FOR ALL OBSERVERS
  - **“Absolute, true time, of itself and of its own nature, flows equably, without relation to anything external”**
- EINSTEIN
  - “A time interval measurement depends on the reference frame the measurement is made”

II. SIMULTANEITY



A stationary observer B_2 on the railway embankment sees the forked lightning strike at two positions q_1 and q_2 simultaneously. The observer B_1 moving with the train undergoing uniform rectilinear motion, sees the lightning strike q_2 a fraction of a second before q_1. The reason for this is the finite signal speed of light.

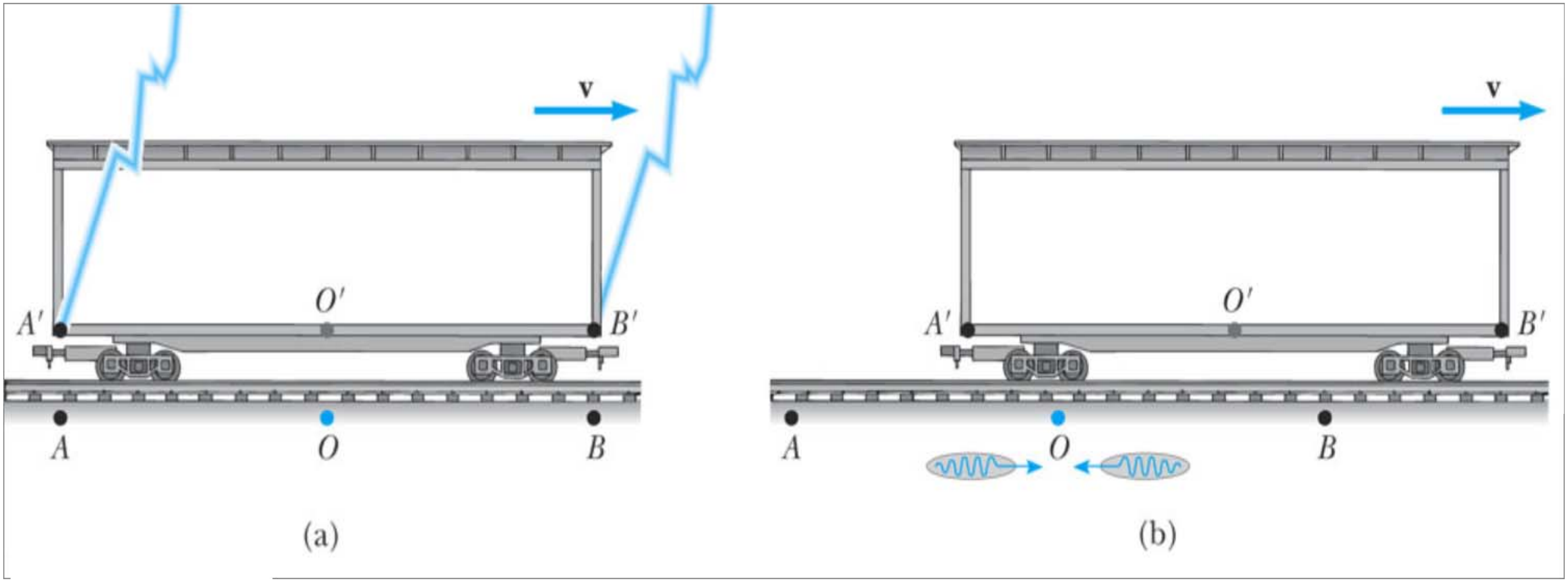
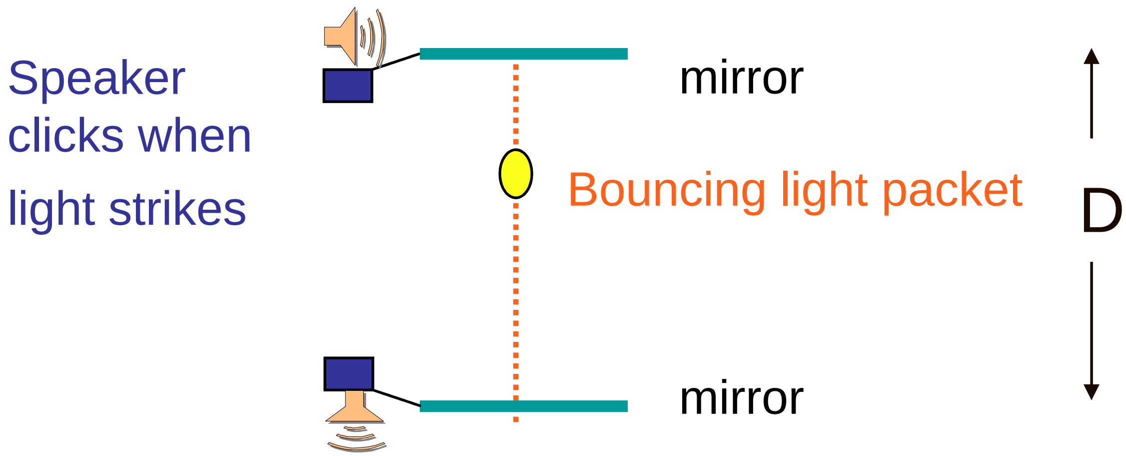


Fig. 1-9, p. 14

II: TIME DILATION

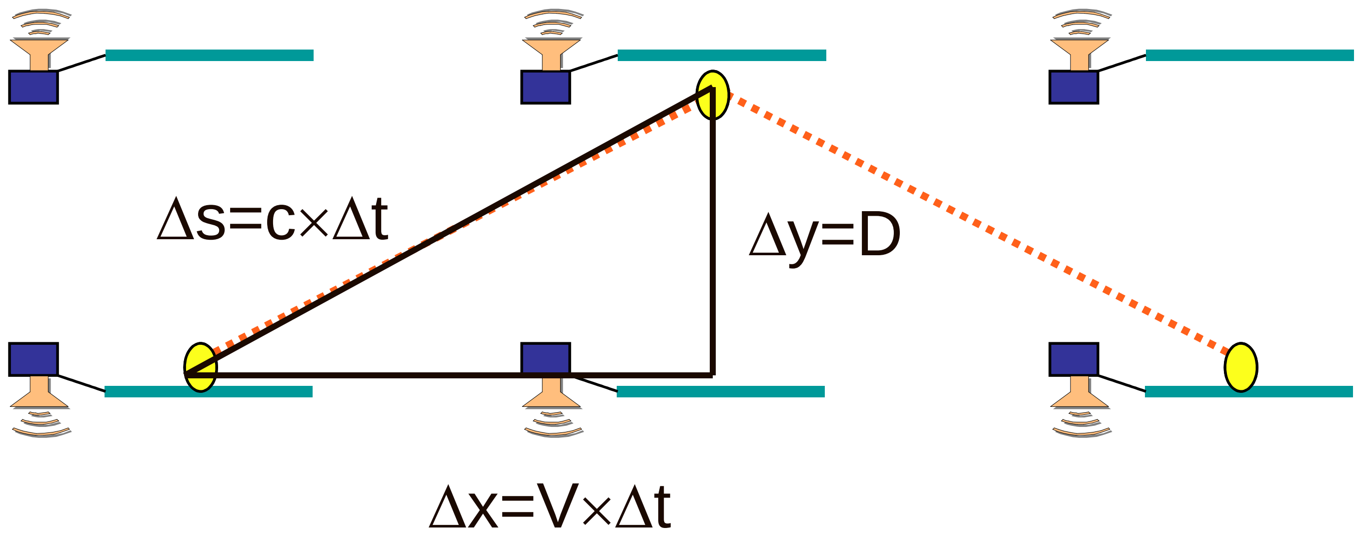
- Imagine building a clock using mirrors and a light beam.



- One "tick" of the clock is the time it takes for light to travel from base to mirror.

$$\Delta T_o = \frac{D}{c}$$

Moving clock



- Now suppose we put the same "clock" on a spaceship that is cruising (at constant velocity, V) past us.
- How long will it take the clock to "tick" when we observe it in the moving spacecraft? Use Einstein's postulates…
- Total distance travelled by light beam is $\Delta s = c \times \Delta t$
- Therefore time $\Delta t = \Delta s / c$
- By Pythagorean theorem, $\Delta s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V\Delta t)^2 + D^2}$
- Can solve to obtain $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- Clock appears to run more slowly!!

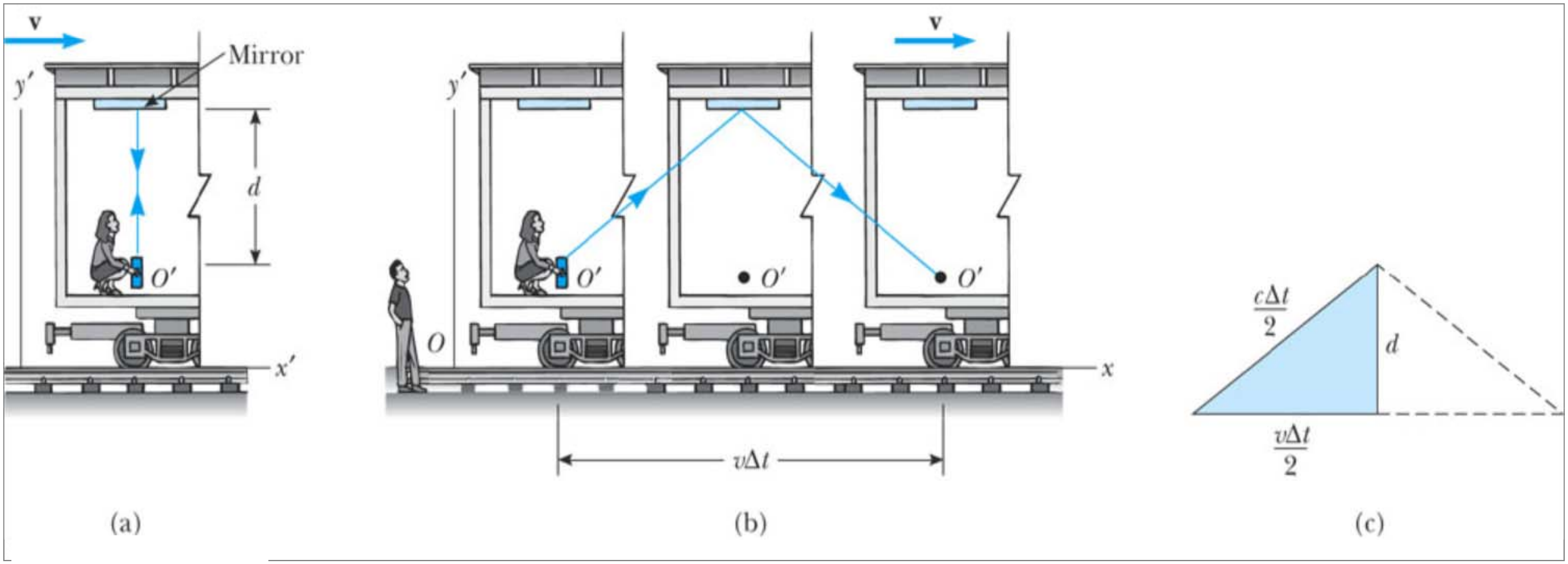


Fig. 1-10, p. 15

Now change the point of view…

- For ground-based observer, clock on spaceship takes longer to "tick" than it would if it were on the ground
- But, suppose there's an astronaut in the spacecraft
  - the inside of the spacecraft is also an inertial frame of reference – Einstein's postulates apply…
  - So, the astronaut will measure a "tick" that lasts

$$\Delta T_o = \frac{D}{c}$$

  - This is just the same time as the "ground" observers measured for the clock their own rest frame
- So, different observers see the clock going at different speeds!
- **Time is not absolute!!**

Time dilation

- This effect called **Time Dilation**.
- Clock always ticks most rapidly when measured by observer in its own rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

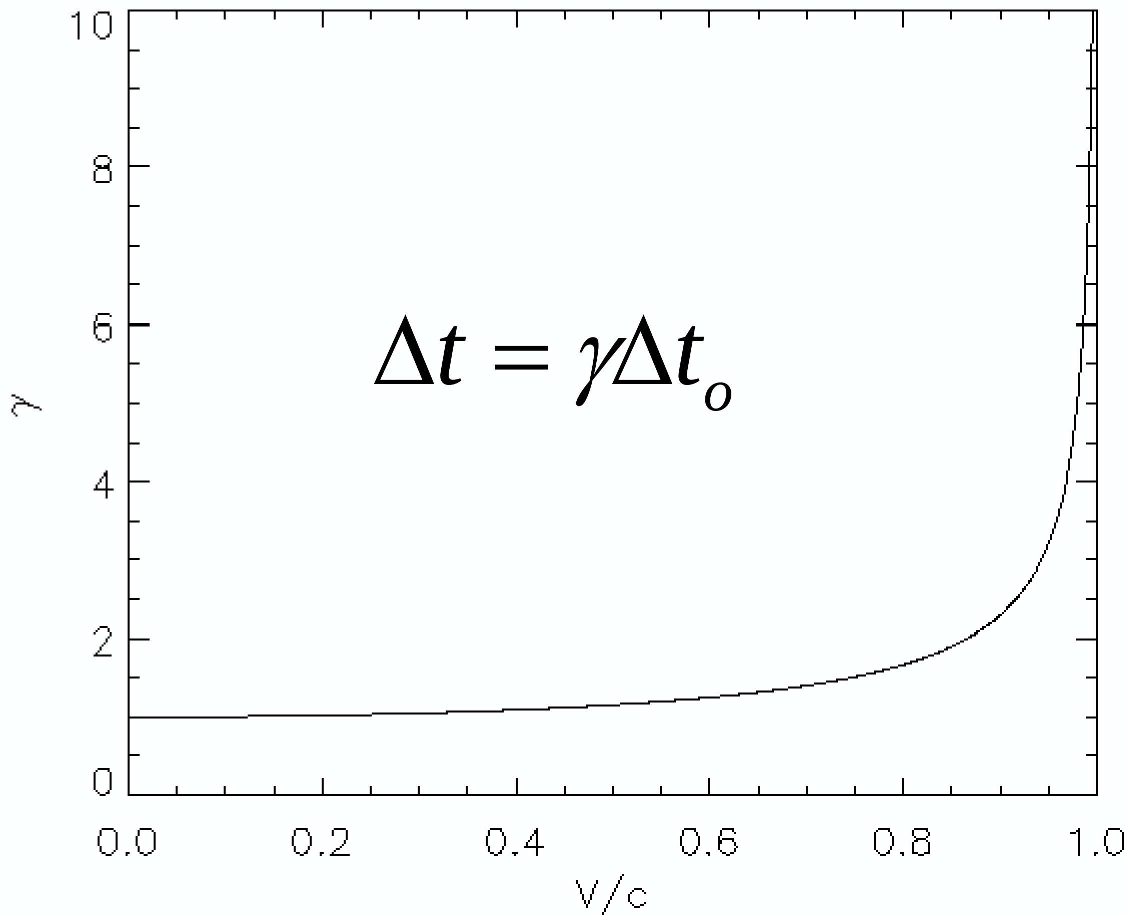
$$\Delta t \div \Delta T_o = \frac{D/c}{\sqrt{1-\mathrm{V}^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1\text{-}\mathrm{V}^2/\mathrm{c}^2}}$$

- This is called the **Lorentz factor,** γ

$$\gamma = \frac{1}{\sqrt{1\text{-}\mathrm{V}^2/\mathrm{c}^2}}$$

Lorentz factor

Notice that the clock ticks longer and as a result the moving person ages less



Lorentz factor goes to infinity when V->c!

But it is very close to 1 for V/c small

Clocks and time

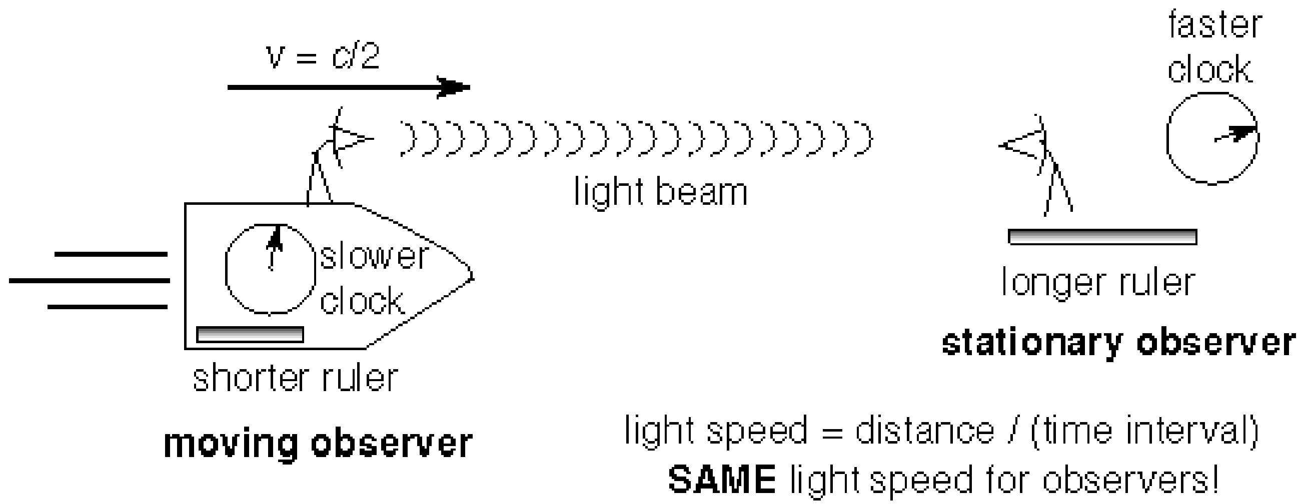
- Does this "time dilation" effect come about because we used a funny clock?
- No, any device that measures time would give the same effect!
- The time interval of an event as measured in its own rest frame is called the ***proper time***
- Note that if the astronaut observed the same "light clock" (or any clock) that was at rest on Earth, it would appear to run slow by the same factor γ , because the dilation factor depends on *relative speed*
- This is called the ***principle of reciprocity***

Why don't we ordinarily notice time dilation?

Some examples of speeds in m/s

- 464 m/s Earth's rotation at the equator.
- 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- 1000 m/s the speed of a typical rifle bullet
- 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- 299,792,458 m/s the speed of light (about 300,000 km/s)

III: LENGTH CONTRACTION



The only way observers in motion relative to each other can *measure* a single light ray to travel the same distance in the same amount of time *relative to their own reference frames* is if their ``meters'' are different and their ``seconds'' are different! Seconds and meters are *relative* quantities.

LENGTH CONTRACTION

- Consider two "markers" in space.
- Suppose spacecraft flies between two markers at velocity V.
- A flash goes off when front of spacecraft passes each marker, so that anyone can record it
- Compare what would be seen by observer at rest w.r.t. markers, and an astronaut in the spacecraft…
- Observer at rest w.r.t. markers says:
  - Time interval is t; distance is $L_o = V \times t$
- Observer in spacecraft says:
  - Time interval is t_o; distance is $L_S = V \times t_o$
- We know from before that $t = t_o \gamma$
- Therefore, $L_S = V \times t_o = V \times t \times (t_o / t) = L_o / \gamma$
- *The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor* γ

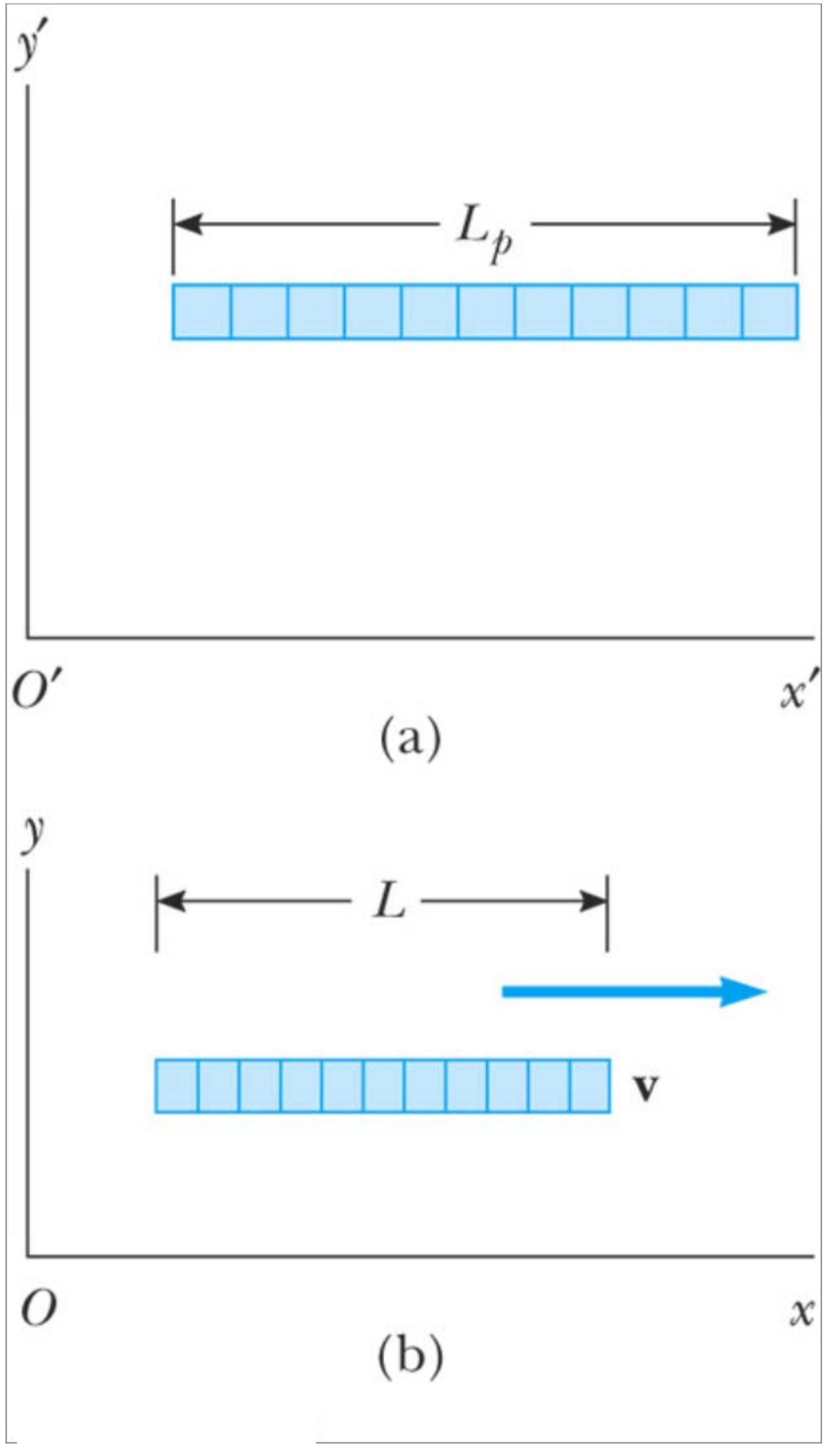


Fig. 1-13, p. 19

- So, moving observers see that objects contract *along the direction of motion*.
- **Length contraction**… also called
  - Lorentz contraction
  - FitzGerald contraction
- Note that there is no contraction of lengths that are perpendicular to the direction of motion
  - Recall M-M experiment: results consistent with *one* arm contracting

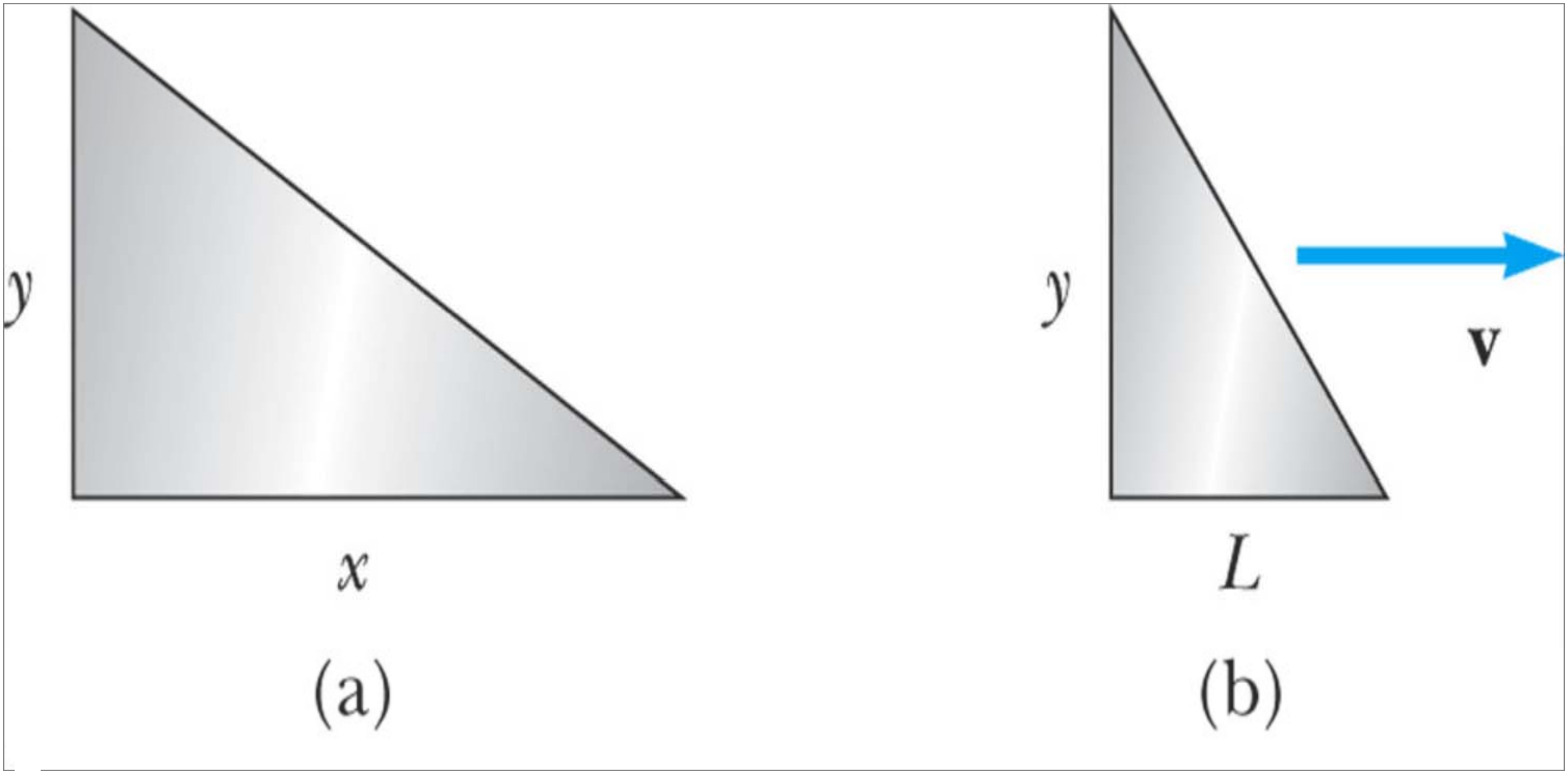


Fig. 1-15, p. 21

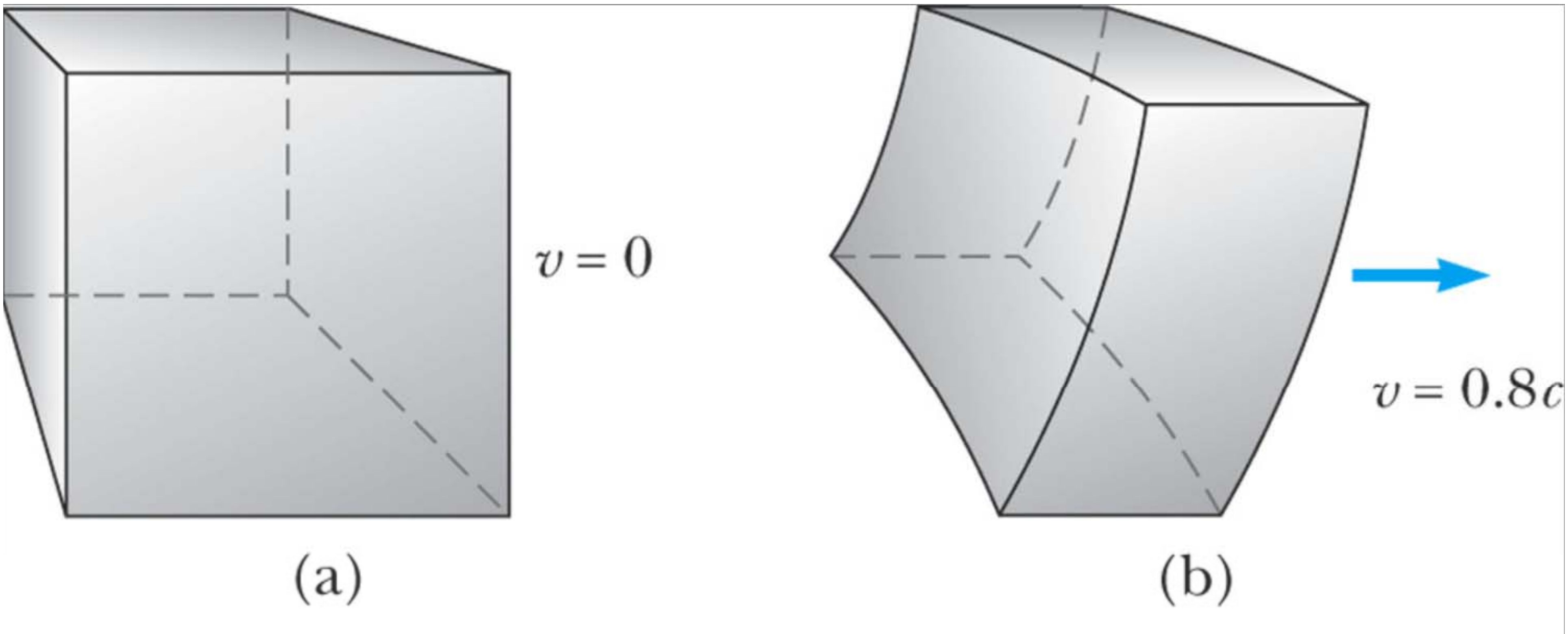


Fig. 1-14, p. 20

Muon Experiment

- The Muon Experiment
  - Muons are created in upper atmosphere from cosmic ray hits
  - Typical muon travel speeds are $0.99995 \times c$, giving $\gamma=100$
  - Half-life of muons in their own rest frame (measured in lab) is t_h= 2 microseconds =0.000002s
  - Travelling at $0.99995 \times c$ for t_h=0.000002s, the muons would go only 600 m
  - But travelling for $\gamma \times t_h$= 0.0002s, the muons can go 60 km
  - *They easily reach the Earth's surface, and are detected!*
  - *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with* $\gamma \times t_h$

Muon experiment, again

- Consider atmospheric muons again, this time from point of view of the muons
  - i.e. think in frame of reference in which muon is at rest
  - Decay time in this frame is 2 μs (2/1000,000 s)
  - How do they get from top of mountain to sea level before decaying?
- From point of view of muon, mountain's height *contracts by factor of γ to 600 m*
  - Muons can then travel reduced distance (at almost speed of light) before decaying.

Examples of time dilation

The Muon Experiment

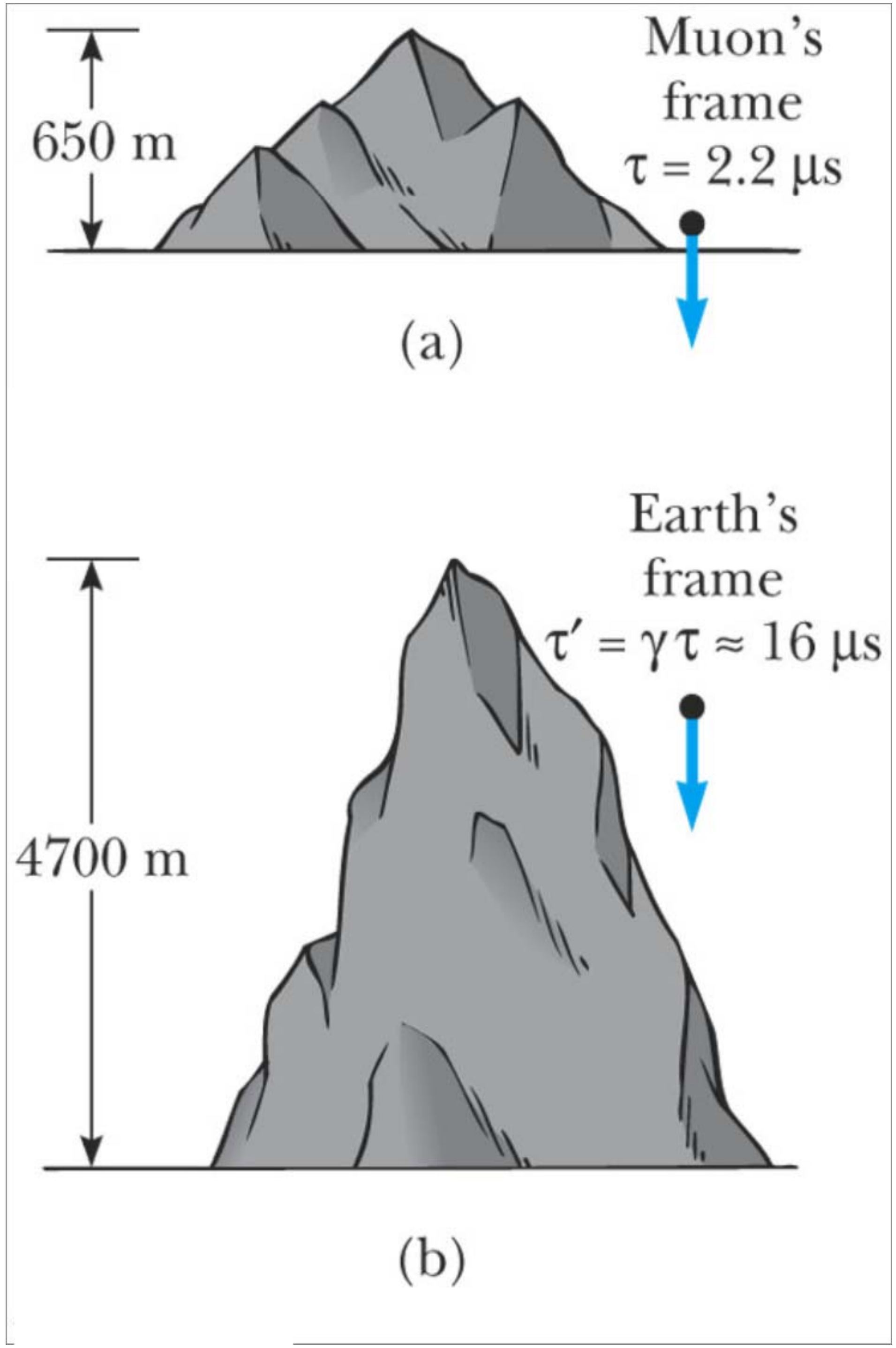


Fig. 1-11, p. 17

New velocity addition law

- Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simple adding/subtracting frame velocity) gives incorrect results for electromagnetism
- Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities
- For a particle measured to have velocity V_p by an observer in a spaceship moving at velocity V_s with respect to Earth, the particle's velocity as measured by observer on Earth is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator ->0 and therefore $V \rightarrow V_p + V_s$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed of light
- This is consistent with everyday experience
- Also notice that if the particle has $V_p = c$ in the spaceship frame, then it has $V_p = c$ in the Earth frame. The speed of light is frame-independent!