

Introduction to Integers

Operation on Integers

Integers:

Integers are the set of whole numbers and their opposites. $\{ \dots -3, -2, -1, 0, 1, 2, 3 \dots \}$ is the set of integers.

Following are some examples of integers: -12, 315, 733, 751, 10, and 121.

Positive Integer:

The numbers greater than zero are called **Positive integers**

Positive integers are represented towards right of zero (0) on a number line. In this collection, 1, 2, 3 ... are said to be positive integers

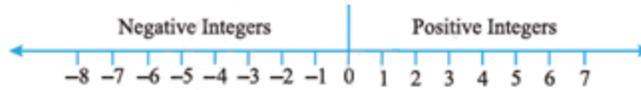
Negative Integer:

A number which is less than zero but not a fraction or a decimal is called a **Negative Integer**. It is represented by putting '-' sign before the number. It is shown to the left of zero on a number line. In this collection, - 1, - 2, - 3 ... are said to be negative integers.

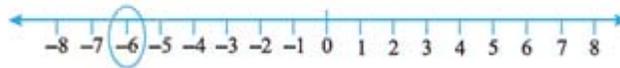
Operation on Integers

Representation of integers on a number line

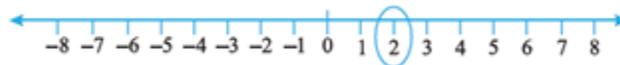
Draw a line and mark some points at equal distance on it as shown in the figure.



Mark a point as zero on it. Points to the right of zero are positive integers and are marked + 1, + 2, + 3, etc. or simply 1, 2, 3 etc. Points to the left of zero are negative integers and are marked - 1, - 2, - 3 etc. In order to mark - 6 on this line, we move 6 points to the left of zero.



In order to mark + 2 on the number line, we move 2 points to the right of zero.



Ordering of integers:

Let us once again observe the integers which are represented on the number line.



We know that $7 > 4$ and from the number line shown above, we observe that 7 is to the right of 4. Similarly, $4 > 0$ and 4 is to the right of 0. Now, since 0 is to the right of -3 so, $0 > -3$. Again, -3 is to the right of -8 so, $-3 > -8$.

Thus, we see that on a number line the number increases as we move to the right and decreases as we move to the left.

Therefore, $-3 < -2$, $-2 < -1$, $-1 < 0$, $0 < 1$, $1 < 2$, $2 < 3$ so on. Hence, the collection of integers can be written as..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5...

Example 1:

Which integers lie between - 10 and - 4? Which is the largest integer and the smallest integer among them?

Solution:

Integers between - 10 and - 6 are - 9, - 8, - 7, -6, -5. The integer - 5 is the largest and - 9 is the smallest.

Addition of Integers:

Let us perform additions with the help of two positive integers.

You add When you have two positive integers like (+5) and (+7) then $(+5) + (+7) = (+12)$ [=5+7]

Example:

Find the addition of

(a) $(+ 10) + (+ 4)$

(b) $(+ 23) + (+ 40)$

Solution:

(a) $(+ 10) + (+ 4) = (10 + 4) = +14$

(b) $(+ 23) + (+ 40) = (23 + 40) = (+63)$

Similarly, you also add when you have two negative integers, but remember that the answer will take a minus (-) sign like $(-8) + (-2) = - (8+2) = -10$.

Example:

Find the solution of the following:

(a) $(- 11) + (- 12)$

(b) $(- 32) + (- 25)$

Solution:

(a) $(- 11) + (- 12) = - (11 + 12) = -23$

(b) $(- 32) + (- 25) = - (32 + 25) = -57$

Now, when you have one positive and one negative integer, you must subtract, but answer will take the sign of the bigger integer.

Example: Fine the addition of

(a) $(+12) + (- 7)$

(b) $(+ 7) + (- 10)$

Solution:

$$(a) (+12) + (-7) = (+12 - 7) = (12 - 7) = 5$$

$$(b) (+7) + (-10) = (+7) + (-10) = (+7 - 10) = (7 - 10) = -3$$

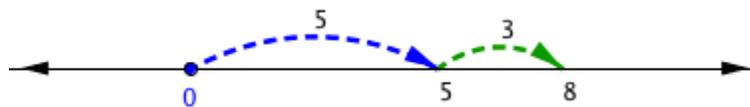
Addition of integers on a number line:

Example 1:

Let us add 5 and 3 on number line.

Solution:

On the number line, we first move 5 steps to the right from 0 reaching 5, and then we move 3 steps to the right of 5 and reach 8. Thus, we get $5 + 3 = 8$

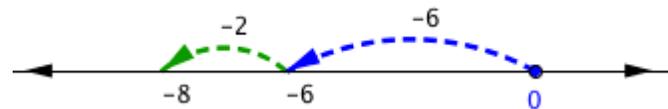


Example 2:

Let us add - 6 and - 2 on the number line.

Solution:

On the number line, we first move 6 steps to the left of 0 reaching - 6, then we move 2 steps to the left of - 6 and reach - 8. Thus, $(- 6) + (- 2) = - 8$



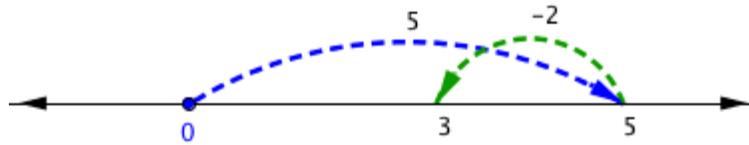
Note: We observe that when we add two positive integers, their sum is a positive integer. When we add two negative integers, their sum is a negative integer.

Example 3:

Find the sum of (+ 5) and (- 2) on the number line.

Solution:

First we move to the right of 0 by 5 steps reaching 5. Then we move 3 steps to the left of 5 reaching 3. Thus, $(+ 5) + (- 2) = 3$

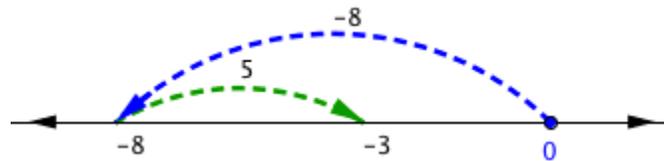


Example 4:

Let us find the sum of $(- 8)$ and $(+ 5)$ on the number line.

Solution:

First we move 8 steps to the left of 0 reaching $- 8$ and then from this point we move 5 steps to the right. We reach the point $- 3$. Thus, $(- 8) + (+5) = - 3$.



Additive Inverse:

The Additive Inverse of a number is the opposite of the number. A number and its opposite add up to give zero. They are called **additive inverse** of each other.

Example:

Find the additive inverse of

- (a) 7
- (b) -2 ?

Solution:

(a) The additive inverse of 7 is $- 7$.
 $7 + (- 7) = 0$

(b) The additive inverse of $- 2$ is 2.
 $- 2 + 2 = 0$

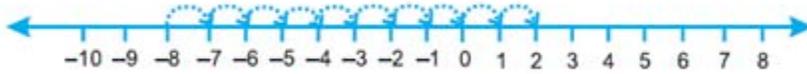
Subtraction of Integers with the help of a Number Line:

Example: 1

Find the value of $- 8 - (-10)$ using number line.

Solution:

$- 8 - (- 10)$ is equal to $- 8 + 10$ as additive inverse of -10 is 10 .
On the number line, from $- 8$ we will move 10 steps towards right.



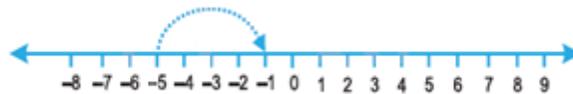
We reach at 2 . Thus, $-8 - (-10) = 2$ Hence, to subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted, to the other integer.

Example: 2

Let us now find the value of $-5 - (-4)$ using a number line.

Solution:

We can say that this is the same as $-5 + (4)$, as the additive inverse of -4 is 4 . We move 4 steps to the right on the number line starting from -5 . We reach at -1 i.e. $-5 + 4 = -1$. Thus, $-5 - (-4) = -1$.



Some more examples of addition and subtraction:

Example 1:

Find the sum of $(- 9) + (+ 4) + (- 6) + (+ 3)$

Solution:

We have,

$$\begin{aligned} & (- 9) + (+ 4) + (- 6) + (+ 3) \\ &= (- 9) + (- 6) + (+ 4) + (+ 3) \\ &= (- 15) + (+ 7) \end{aligned}$$

$$= - 8$$

Example 2:

Find the value of $(30) + (- 23) + (- 63) + (+ 55)$

Solution:

$$(30) + (+ 55) + (- 23) + (- 63)$$

$$= 85 + (- 86)$$

$$= - 1$$

Example 3:

Find the sum of $(- 10)$, (92) , (84) and $(- 15)$

Solution:

$$(- 10) + (92) + (84) + (- 15)$$

$$= (- 10) + (- 15) + 92 + 84$$

$$= (- 25) + 176 = 151$$

Example 4:

Subtract $(- 4)$ from $(- 10)$

Solution:

$$(- 10) - (- 4)$$

$$= (- 10) + (\text{additive inverse of } - 4)$$

$$= -10 + 4 = - 6$$

Example 5:

Subtract $(+ 3)$ from $(- 3)$

Solution:

$$(- 3) - (+ 3)$$

$$= (-3) + (\text{additive inverse of } +3)$$

$$= (-3) + (-3) = -6$$