Introduction about Algebra

Use of Variables in Common Rules

Expression with Variable

Equation

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Algebra

Algebra is a branch of mathematics that deals in representing numbers through variables.

Algebra also deals with symbols, relations, functions, and equations.

For Example:

5 + a = 12
Here, the variable 'a' represents a whole number and it can be solved to find its actual value. 'a' must be equal to 7 as 5 + 7 = 12.

Matchstick Patterns:

Sonal and Minal are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Sonal takes two matchsticks and forms the letter L as shown in Fig (a)

Then Minal also picks two sticks, forms another letter L and puts it next to the one made by Sonal in Fig (b). Then Sonal adds one more L and this goes on as shown by the dots in Fig (c). Their friend Mohan comes in. He looks at the pattern. Mohan always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Sonal and Minal are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	….
Number of matchstick required	2	4	6	8	10	12	14	16	….

Mohan gets the answer to his question from the Table 1; 7Ls require 14 matchsticks. While writing the table, Sonal realizes that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = 2 × number of Ls.

For convenience, let us write the letter n for the number of

Ls. If one L is made, n = 1; if two Ls are made, n = 2 and so on; thus, n can be any natural number 1, 2, 3, 4, 5 ... We then write,
Number of matchsticks required = 2 × n.
Instead of writing 2 × n, we write 2n. Note that 2n is same as 2 × n.

Sonal tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.
Thus, for n = 1, the number of matchsticks required = 2 × 1 = 2
For n = 2, the number of matchsticks required = 2 × 2 = 4
For n = 3, the number of matchsticks required = 2 × 3 = 6 etc.
These numbers agree with those from Table 1.

The Idea of a Variable:

Variables are (usually) letters or other symbols that represent unknown numbers or values. The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was:

Number of matchsticks required = 2n

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4...

In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

n is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4 ... We wrote the rule for the number of matchsticks required using the variable n.

Note: One may use any letter as m, l, p, x, y, z etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3.
It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But n in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4...

Examples of Variable:

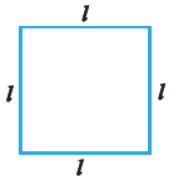
The following are examples of algebraic expressions and equations containing variables.
2x + 5 = 10, the variable here is x
7y + 10 = 24, the variable here is y
a + b, the variables here are a and b

Use of Variables in Common Rules

Rules from geometry:

Perimeter of a square:

A square has 4 sides and they are equal in length.

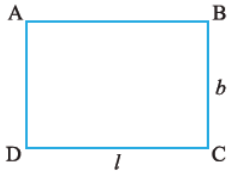


Therefore, The perimeter of a square = Sum of the lengths of the sides of the square
= 4 times the length of a side of the square
= 4 × **l** = 4**l**.

Thus, we get the rule for the perimeter of a square. The use of the variable **l** allows us to write the general rule in a way that is concise and easy to remember.
We may take the perimeter also to be represented by a variable, say **p**.
Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, **p** = 4**l**

Perimeter of a rectangle:

The rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length.



Thus, in the rectangle ABCD, let us denote by **l**, the length of the sides AB or CD and, by **b**, the length of the sides AD or BC.

Therefore, Perimeter of a rectangle = length of AB + length of BC + length of CD + length of AD
= 2 × length of CD + 2 × length of BC = 2**l** + 2**b**

The rule, therefore, is that the perimeter of a rectangle = 2**l** + 2**b** where, **l** and **b** are respectively the length and breadth of the rectangle. If we denote the perimeter of the rectangle by the variable **p**, the rule for perimeter of a rectangle becomes **p = 2l + 2b**

Note: Here, both **l** and **b** are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

Rules from arithmetic:

Commutativity of addition of two numbers:

The Commutative Property of Addition states that changing the order of addends does not change the sum, i.e. Let a and b be two variables which can take any number value. Then,

$$\mathbf{a + b = b + a}$$

Once we write the rule this way, all special cases are included in it. If **a** = 7 and **b** = 8, we get 7 + 8 = 8 + 7. If **a** = 12 and **b** = 34, we get 12 + 34 = 34 + 12 and so on.

Commutativity of multiplication of two numbers:

The Commutative Property of Multiplication states that changing the order of factors does not change the product, i.e. Let a and b be two variables which can take any number value. Then,

$$\mathbf{a \times b = b \times a.}$$

For example:

3 × 9 = 9 × 3 and 37 × 73 = 73 × 37 follow from the general rule.

Distributivity of numbers:

Distributive Property states that the product of a number and a sum is equal to the sum of the individual products of the addends and the number. Let *a*, *b* and *c* be three variables each of which can take any number.
That is,

$$\mathbf{a \times (b + c) = a \times b + a \times c.}$$

For example:

7 × 38 = 7 × (30 + 8) = 7 × 30 + 7 × 8 = 210 + 56 = 266

Expression with Variable

An **expression** consists of terms that are written with arithmetical signs between them such as the addition, subtraction, division and multiplication signs.

For example:

$2n$, $5m$, $x + 10$, $x - 3$ etc. where the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

Example: 1
How these expressions have been formed?

(a) y + 5
(b) r + 25
(c) 10 y + 7
(d) – 5 q
(e) x/3

Solution:

(a) 5 added to y
(b) 25 added to r
(c) y multiplied by 10 and then 7 added to the product
(d) q multiplied by –5
(e) x divided by 3

Examples: 2

Give expressions in the following cases

(a) n multiplied by 2 and 1 subtracted from the product.
(b) a multiplied by 10.
(c) 7 subtracted from – m
(d) 7 subtracted from p
(e) y is multiplied by – 8 and then 5 is added to the result.

Solution:

(a) 2n -1
(b) 10a
(c) –m-7
(d) P-7
(e) -8y +5

Equation

An Equation is a mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

Example:

The following are some examples of equation.

$10 + 2 = 12$
$4a - 3 = 1$
$5x + 8 = 40$

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example:

The statement 2n is greater than 10, i.e. $2n > 10$ is not an equation. Similarly, the statement 2n is smaller than 10 i.e. $2n < 10$ is not an equation. Also, the statements $(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Example:

Check which of the following are equations (with a variable)?

(a) $2n + 1 = 11$
(b) $(3/2)\ q < 5$
(c) $7 - x = 5$
(d) $(t - 7) > 5$

Solution:

(a) **Yes,** $2n + 1 = 11$ are an equation because it has a variable n and equal to sign.
(b) **No,** $(3/2)\ q < 5$ are not an equation because it has a variable q but not equal to sign.
(c) **Yes,** $7 - x = 5$ are an equation because it has a variable x and equal to sign.
(d) **No,** $(t - 7) > 5$ are not an equation because it has a variable t but not equal to sign.

Solution of an Equation:

The value of the variable in an equation which satisfies the equation is called a solution to the equation.

For example:
Let us take the equation $x - 3 = 11$
This equation is satisfied by $x = 14$, because for $x = 14$,
LHS of the equation $= 14 - 3 = 11 =$ RHS.

Example:

Complete the table and Find the solution to the equation $x - 7 = 3$.

X	3	2	6	10	16	5
X -7						

Solution:

X	3	2	6	10	16	25
X -7	-4	-5	-1	3	9	18

We have $x - 7 = 3$,
Add 7 both sides, we get
$X - 7 + 7 = 3 + 7$
$X = 3 + 7 = 10$
$X = 10$

Hence, the solution of the equation $x - 7 = 3$ is $x = 10$.