

Introduction to Fraction

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Introduction to Fraction

Fraction

A fraction is a number that represents part of a whole. A fraction is written in the form p/q , where $q \neq 0$.

The top number of a fraction is called the **numerator**. The bottom number of a fraction is called the **denominator**.

Example:

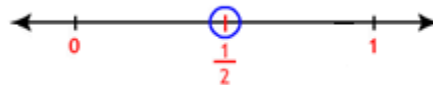
$\frac{1}{2}$ is a fraction, with numerator 1 and denominator 2.

Fractions on a number line:

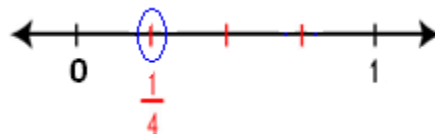
Let us draw a number line and try to mark $\frac{1}{2}$ on it.



We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1. Since we have to show $\frac{1}{2}$ we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$.



Now, we want to show $\frac{1}{4}$ on a number line. We divide the length between 0 and 1 into 4 equal parts and show one part as $\frac{1}{4}$.



Similarly, we can also show $\frac{2}{4}$ and $\frac{3}{4}$ on a number line as shown in the figure below.



Kinds of Fraction

Proper Fractions:

A fraction having the numerator less than the denominator is called a **Proper Fraction**. The value of a proper fraction is always less than 1.

For Example:

$\frac{2}{5}$, $\frac{4}{5}$, $\frac{7}{11}$, $\frac{2}{13}$ are proper fractions.

Improper Fraction:

An improper fraction is a fraction in which the number in the numerator is greater than or equal to the number in the denominator.

For Example:

$\frac{17}{15}$, $\frac{9}{1}$, $\frac{32}{12}$, $\frac{4}{2}$ are improper fraction.

Mixed Fraction:

A Mixed Fraction is a number with a combination of an integer and a proper fraction. Mixed fraction is also called as mixed number.

The mixed fraction can also be written as Quotient = Remainder/Divisor.

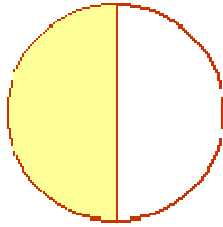
For Example:

$2\frac{3}{4}$, $5\frac{7}{8}$, $3\frac{9}{10}$ are mixed fractions.

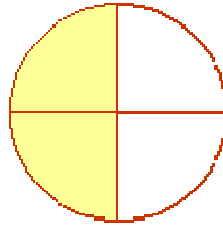
Equivalent Fractions:

Equivalent Fractions of a fraction are those fractions whose numerator and denominator are in the same ratio as that of the original fraction.

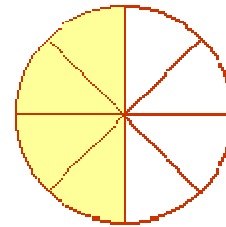
Equivalent fraction can be generated by multiplying or dividing the numerator and denominator by the same number.



1 out of 2



2 out of 4



4 out of 8

Example:

In the given figure, shaded parts of the figures represent the equivalent fractions

$$\text{as } \frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

Understanding equivalent fractions:

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

For example:

$$1/5 = (1 \times 2) / (5 \times 2) = 2/10.$$

$$\text{Similarly, } 2/3 = (2 \times 3) / (3 \times 3) = 6/9.$$

Now we see another way,

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

For example:

$$15/35 = (15 \div 5) / (35 \div 5) = 3/7$$

$$\text{Similarly, } 16/20 = (16 \div 4) / (20 \div 4) = 4/5$$

Simplest Form of a Fraction:

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

Example:

Find the simplest form of 10/18?

Solution:

We have,

$$36/54 = (36 \div 2) / (54 \div 2) = 18/27$$

But 18 and 27 also have common factors other than one.
The common factors are 1, 3, 9; the highest is 9.

$$\text{Therefore, } 18/27 = (18 \div 9) / (27 \div 9) = 2/3$$

Now 2 and 3 have no common factor except 1; we say that the fraction $2/3$ is in the simplest

The shortest way, The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

For example:

Find the simplest form of $26/39$?

Solution:

We have, $26/39$

The HCF of 26 and 39 is 13.

So, $(26 \div 13) / (39 \div 13) = 2/3$. The fraction $2/3$ is in the lowest form. Thus, HCF helps us to reduce a fraction to its lowest form.

Like Fractions:

The different fractions with the same denominator are **Like Fractions**.

Example:

$3/7$, $5/7$, $11/7$, $8/7$ are like fractions, as they have the same denominator 7.

Unlike Fractions:

Fractions whose denominators are not the same are called **unlike fractions**.

Example:

$10/7$, $5/4$, $11/8$, $5/2$ are unlike fractions, as they have the different denominator.

Comparing Fractions

Comparing like fractions:

When comparing two fractions with like denominators, the larger fraction is the one with the greater numerator. Let's look at some examples of comparing fractions with like denominators.

Example 1:

Compare between $\frac{2}{5}$ and $\frac{8}{5}$.

Solution:

Given fractions are like fractions, so the fraction with greater numerator is larger. Hence, $\frac{8}{5}$ is greater than $\frac{2}{5}$.

Example 2:

Find among the given two fraction which is greater $\frac{24}{87}$ and $\frac{10}{87}$.

Solution:

Given fractions have same denominator, so the fraction with greater numerator is larger.
Hence, $\frac{24}{87} > \frac{10}{87}$

Example 3:

Compare between $\frac{23}{10}$ and $\frac{13}{10}$.

Solution:

Given fractions are like fractions, so the fraction with greater numerator is larger.
Hence, $\frac{23}{10} > \frac{13}{10}$

Example 4: Find among the given two fraction which is greater $\frac{11}{90}$ and $\frac{77}{90}$.

Solution:

Given fractions have same denominator, so the fraction with greater numerator is larger.
Hence, $\frac{77}{90}$ is greater than $\frac{11}{90}$.

Comparing unlike fractions (with same numerator)

When you need to compare unlike fraction with same numerator, the fraction with smallest denominator is greatest. Let's look at some examples of comparing fractions with same numerator

Example 1:

Compare $15/6$ and $15/8$

Solution:

The given fractions have same numerator i.e. 15, so we compare them on the basis of their denominator.

The fraction which have smallest denominator is greatest.

In this, denominator 6 is smaller than denominator 8.

Hence, $15/6 > 15/8$

Example 2:

Compare between $9/8$ and $9/2$

Solution:

The given fractions have same numerator i.e. 9, so we compare them on the basis of their denominator.

The fraction which have smallest denominator is greatest.

In this, denominator 2 is smaller than denominator 8.

Hence, $9/2 > 9/8$

Example 3:

Compare $77/32$ and $77/23$

Solution:

The given fractions have same numerator i.e. 77, so we compare them on the basis of their denominator.

The fraction which have smallest denominator is greatest.

In the given example denominator 23 is smaller than denominator 32.

Hence, $77/23 > 77/32$

Example 4:

Compare between $243/100$ and $243/150$

Solution:

The given fractions have same numerator i.e. 243, so we compare them on the basis of their denominator.

The fraction which have smallest denominator is greatest.

In this, denominator 100 is smaller than denominator 150.

Hence, $243/100 > 243/150$

Comparing unlike fractions (with different numerator and denominator):

When you need to compare two Unlike Fractions whose numerators are different; First find equivalent fractions of both the given unlike fractions with a common denominator. Then compare equivalent fractions and the fraction with larger numerator is the biggest.

Example 1:

Compare $1/3$ and $2/5$.

Solution:

The given unlike fractions have different numerators i.e. 1 and 2

So, first find a common denominator with the help of LCM of denominator i.e. 3 and 5 and we get; LCM of 3 and 5 = 15.

Now, find equivalent fractions with common denominator(15) of given unlike fraction;

Equivalent Fraction of $1/3 = (1 \times 5) / (3 \times 5) = 5/15$

And, Equivalent Fraction of $2/5 = (2 \times 3) / (5 \times 3) = 6/15$

Now, compare equivalent fractions i.e. $5/15$ and $6/15$;

As numerator 6 is larger than numerator 5 of given fractions,

So $6/15$ is greater than $5/15$

or corresponding fractions; $2/5$ is greater than $1/3$.

Example 2:

Compare $7/6$ and $1/15$.

Solution:

The given unlike fractions have different numerators i.e. 7 and 1,

So, first find a common denominator with the help of LCM of denominator i.e. 6 and 15 and we get;

LCM of 6 and 15 = 30

Now, find equivalent fractions with common denominator(30) of given unlike fraction;

Equivalent Fraction of $7/6 = (7 \times 5) / (6 \times 5) = 35/30$

And, Equivalent Fraction of $1/15 = (1 \times 2) / (15 \times 2) = 2/30$

Now, compare equivalent fractions i.e. $35/30$ and $2/30$;

As numerator 35 is larger than numerator 2 of given fractions,
So $35/30$ is greater than $2/30$
or corresponding fractions; $7/6$ is greater than $1/15$.

Example 3:

Compare $7/10$ and $3/4$.

Solution:

The given unlike fractions have different numerators i.e. 7 and 4,
So, first find a common denominator with the help of LCM of denominator i.e. 10 and 4 and we get;

LCM of 10 and 4 = 20.

Now, find equivalent fractions with common denominator(20) of given unlike fraction;

Equivalent Fraction of $7/10 = (7 \times 2) / (10 \times 2) = 14/20$

And, Equivalent Fraction of $3/4 = (3 \times 5) / (4 \times 5) = 15/20$

Now, compare equivalent fractions i.e. $14/20$ and $15/20$;

As numerator 15 is larger than numerator 14 of given fractions,

So $15/20 > 14/20$

or corresponding fractions; $3/4$ is greater than $7/10$

Addition and Subtraction of Fraction

Addition of fractions (with Same Denominator):

When we have to add fractions with same denominators, we simply add the numerators and denominator remains same.

Example: 1

Add fractions $1/10$ and $5/10$

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
 $(1+5) / 10 = 6 / 10 = 3 / 5$ (in Lowest Term)

Example: 2

Add $10/3$ and $22/3$

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
 $(10+22) / 3 = 32 / 3$.

Example: 3

Add $2/101$ and $4/101$

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
 $(2+4) / 101 = 6 / 101$.

Example: 4

Add $25/39$ and $40/39$

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
 $(25+40) / 39 = 65 / 39$.

Addition of Fraction (with Different Denominators):

It involves following steps:

Step 1 = LCM of Denominators of given Fraction

Step 2 = Divide LCM (obtained in step 1) with the denominators of given fractions.

Step 3 = Multiply quotient (obtained from step 2) with the numerators of given fractions.

Example:

Add $(2/7)$ and $(3/5)$.

Solution:

The addition proceeds as $(2/7) + (3/5)$

LCM of denominators 7 and 5 = 35

Now, divide the LCM by denominator & multiply its quotient with numerators and we get:

$$= [(2 \times 5) + (3 \times 7)] / 35$$

$$= (10 + 21) / 35$$

$$= 31 / 35$$

Subtraction of Fractions (with Same Denominator):

When we have to subtract fractions with same denominators (or subtract like fractions). We simply subtract the numerators and denominator remains same.

Example: 1

Subtract $10/2$ from $15/2$.

Solution:

$$(15/2) - (10/2)$$

In the given fractions the denominators are same, so subtraction proceeds as:

Subtract the numerators and denominator is kept common and we get:

$$= (15 - 10) / 2$$

$$= 5 / 2.$$

Example: 2

Subtract $10/11$ from $8/11$.

Solution:

$$(8/11) - (10/11)$$

In the given fractions the denominators are same, so subtraction proceeds as:

Subtract the numerators and denominator is kept common and we get:

$$= (8 - 10) / 11$$

$$= -2 / 11.$$

Example: 3

Solve $(29/100) - (28/100)$.

Solution:

In the given fractions the denominators are same, so subtraction proceeds as:

Subtract the numerators and denominator is kept common and we get:

$$= (29 - 28) / 100$$

$$= 1 / 100$$

Subtraction of Fractions (with Different Denominator):

To subtract fractions with different denominators, we must follow these steps:

Step 1 = Find equivalent fractions of given fraction with common denominator

Step 2 = Follow the process of subtraction of fractions with same denominator

Example 1:

Subtract $3/4$ from $5/6$.

Solution:

We need to find equivalent fractions of $3/4$ and $5/6$, which have the same denominator. This denominator is given by the LCM of 4 and 6. The required LCM of 4 and 6 is 12.

$$\text{Therefore, } 5/6 - 3/4 = (5 \times 2) / (6 \times 2) - (3 \times 3) / (4 \times 3) = 10/12 - 9/12 = 1/12$$

Example 2:

Subtract $2/5$ from $5/7$.

Solution:

We need to find equivalent fractions of $2/5$ and $5/7$, which have the same denominator. This denominator is given by the LCM of 5 and 7. The required LCM of 5 and 7 is 35.

Therefore, $5/7 - 2/5 = (5 \times 5) / (7 \times 5) - (2 \times 7) / (5 \times 7) = 25/35 - 14/35 = 11/35$.

Addition of Mixed Fraction:

Example: 1

Add $2(4/5)$ and $3(5/6)$.

Solution:

$$2(4/5) + 3(5/6) = 2 + 4/5 + 3 + 5/6 = 5 + 4/5 + 5/6.$$

$$\begin{aligned} \text{Now } 4/5 + 5/6 &= (4 \times 6) / (5 \times 6) + (5 \times 5) / (6 \times 5) \\ &= 24/30 + 25/30 = 49/30 = (30 + 19) / 30 = 1 + 19/30. \end{aligned}$$

(Since LCM of 5 and 6 = 30)

$$\begin{aligned} \text{Thus, } 5 + 4/5 + 5/6 &= 5 + 1 + 19/30 = 6 + 19/30 \\ &= 6(19/30) \end{aligned}$$

$$\text{And, therefore, } 2(4/5) + 3(5/6) = 6(19/30)$$

Example: 2

Add $4(2/3)$ and $3(1/4)$.

Solution:

$$4(2/3) + 3(1/4) = 4 + 2/3 + 3 + 1/4 = 7 + 2/3 + 1/4.$$

$$\begin{aligned} \text{Now } 2/3 + 1/4 &= (2 \times 4) / (3 \times 4) + (1 \times 3) / (4 \times 3) \\ &= 8/12 + 3/12 = 11/12 \end{aligned}$$

$$\text{Thus, } 7 + 11/12 = 7(11/12)$$

$$\text{And, therefore, } 4(2/3) + 3(1/4) = 7(11/12)$$

Subtraction of Mixed Fraction:

Example: 1

Subtract $1\frac{3}{4}$ from $2\frac{1}{5}$.

Solution:

$$\begin{aligned} &2\frac{1}{5} - 1\frac{3}{4} \\ &= (2 + \frac{1}{5}) - (1 + \frac{3}{4}) \\ &= [(2 \times 5) + 1]/5 - [(4 \times 1) + 3]/4 \\ &\text{Now, } (10+1)/5 - (4 + 3)/4 = 11/5 - 7/4 \\ &= (11 \times 4) / (5 \times 4) - (7 \times 5) / (4 \times 5) \\ &= 44/20 - 35/20 = 9/20 \\ &\text{And, therefore, } 2\frac{1}{5} - 1\frac{3}{4} = (9/20) \end{aligned}$$

Example: 2

Subtract $2\frac{2}{5}$ from $3\frac{5}{6}$.

Solution:

$$\begin{aligned} &3\frac{5}{6} - 2\frac{2}{5} \\ &= 3\frac{5}{6} - 2\frac{2}{5} = 3 + \frac{5}{6} - 2 - \frac{2}{5} = 1 + \frac{5}{6} - \frac{2}{5}. \\ &\text{Now, } (\frac{5}{6}) - (\frac{2}{5}) = (5 \times 5) / (6 \times 5) - (2 \times 6) / (5 \times 6) \\ &= 25/30 - 12/30 = 13/30 \\ &\text{Thus, } 1 + (\frac{5}{6} - \frac{2}{5}) = 1 + 13/30 \\ &\text{And, therefore, } 3\frac{5}{6} - 2\frac{2}{5} = 1\frac{13}{30} \end{aligned}$$