Wave

A wave is a disturbance or oscillation that travels through spacetime, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium—that is, with little or no associated mass transport. They consist, instead, of oscillations or vibrations around almost fixed locations. Waves are described by a wave equation which sets out how the disturbance proceeds over time. The mathematical form of this equation varies depending on the type of wave.

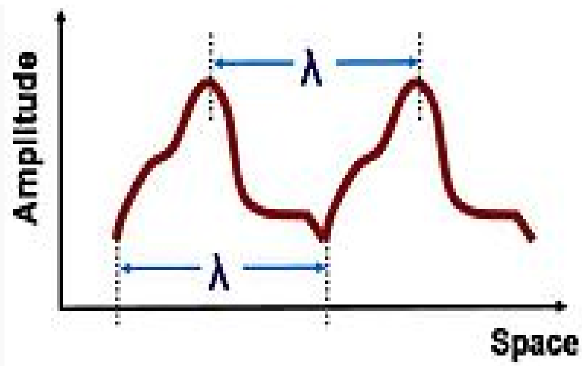
There are two main types of waves. Mechanical waves propagate through a medium, and the substance of this medium is deformed. The deformation reverses itself owing to restoring forces resulting from its deformation. For example, sound waves propagate via air molecules colliding with their neighbors. When air molecules collide, they also bounce away from each other (a restoring force). This keeps the molecules from continuing to travel in the direction of the wave.

The second main type of wave, electromagnetic waves, do not require a medium. Instead, they consist of periodic oscillations in electrical and magnetic fields generated by charged particles, and can therefore travel through a vacuum. These types of waves vary in wavelength, and include radio

waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays.

Wave equation

Consider a traveling transverse wave (which may be a pulse) on a string (the medium). Consider the string to have a single spatial dimension. Consider this wave as traveling



Wavelength *λ*, can be measured between any two corresponding points on a waveform

- **in the x direction in space. E.g., let the positive x direction be to the right, and the negative x direction be to the left.**
- **with constant amplitude u**
- **with constant velocity v, where v is**
  - **independent of wavelength (no dispersion)**
  - **independent of amplitude (linear media, not nonlinear).**
- **with constant waveform, or shape**

This wave can then be described by the two-dimensional functions

$u(x,\ t) = F(x - v\ t)$ **(waveform F traveling to the right)**
$u(x,\ t) = G(x + v\ t)$ **(waveform G traveling to the left)**

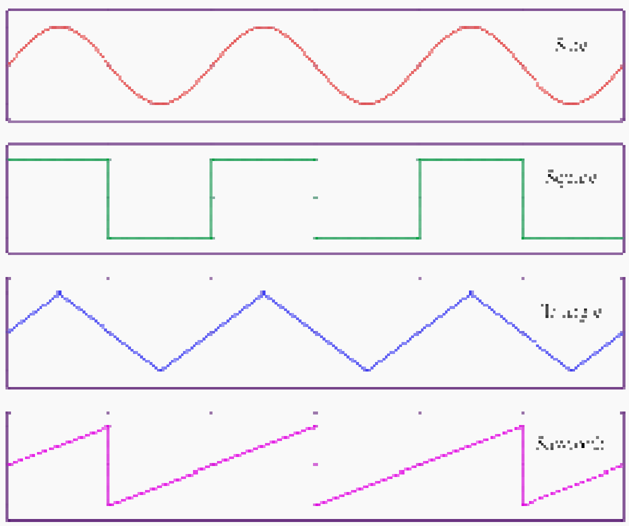
or, more generally, by d'Alembert's formula:

$$u(x,t) = F(x - vt) + G(x + vt).$$

representing two component waveforms F and G traveling through the medium in opposite directions. This wave can also be represented by the partial differential equation

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$

Wave forms



Sine, square, triangle and sawtooth waveforms.

The form or shape of *F* in d'Alembert's formula involves the argument *x* − *vt*. Constant values of this argument correspond to constant values of *F*, and these constant values occur if *x* increases at the same rate that *vt* increases. That is, the wave shaped like the function *F* will move in the positive *x*-direction at velocity*v* (and *G* will propagate at the same speed in the negative *x*-direction).

In the case of a periodic function *F* with period *λ*, that is, *F*(*x* + *λ* − *vt*) = *F*(*x* − *vt*), the periodicity of *F* in space means that a snapshot of the wave at a given time *t*finds the wave varying periodically in space with period *λ* (the wavelength of the wave). In a similar fashion, this periodicity of *F* implies a periodicity in time as well:*F*(*x* − *v(t* + *T)*) = *F*(*x* − *vt*) provided *vT* = *λ*, so an observation of the wave at a fixed location *x* finds the wave undulating periodically in time with period *T* = *λ*/*v*.

Amplitude and modulation

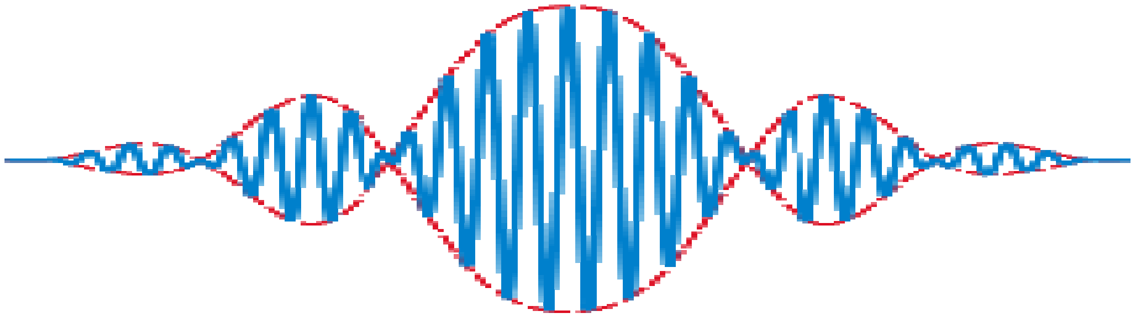


Illustration of the *envelope* (the slowly varying red curve) of an amplitude-modulated wave. The fast varying blue curve is the *carrier* wave, which is being modulated.

The amplitude of a wave may be constant (in which case the wave is a *c.w.* or ***continuous wave***), or may be ***modulated*** so as to vary with time and/or position. The outline of the variation in amplitude is called the ***envelope*** of the wave. Mathematically, the modulated wavecan be written in the form:

$$u(x,\ t) = A(x,\ t)\sin(kx - \omega t - \phi)\ ,$$

where $A(x,\ t)$ is the amplitude envelope of the wave, k is the ***wavenumber*** and ϕ is the***phase***. If the group velocity v_g (see below) is wavelength-independent, this equation can be simplified as:

$$u(x,\ t) = A(x - v_g\ t)\sin(kx - \omega t + \phi)\ ,$$

showing that the envelope moves with the group velocity and retains its shape. Otherwise, in cases where the group velocity varies with wavelength, the pulse shape changes in a manner often described using an ***envelope equation***